Trend Fundamentals and Exchange Rate Dynamics*

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Abstract

We estimate a multivariate unobserved components stochastic volatility model to explain the dynamics of a panel of six exchange rates against the US Dollar. The empirical model is based on the assumption that both countries' monetary policy strategies may be well described by Taylor rules with a time-varying inflation target, a time-varying natural rate of unemployment, and interest rate smoothing. Compared to the existing literature, our model simultaneously provides estimates of the latent components included in a typical Taylor rule specification and the model-based real exchange rate. Our estimates closely track major movements along with important time series properties of real and nominal exchange rates across all currencies considered, outperforming a benchmark model that does not account for changes in trend inflation and trend unemployment. More precisely, the proposed approach improves upon competing models in tracking the actual evolution of the real exchange rate in terms of simple correlations while it appreciably improves upon simpler competitors in terms of matching the persistence of the real exchange rate.

JEL classification: F31, E52, F41, C5, E31.

Keywords: Exchange rate models, trend inflation, natural rate of unemployment, Taylor rule, unobserved components stochastic volatility model.

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1 Introduction

To what extent do economic fundamentals explain exchange rate movements? Following the seminal work by Meese and Rogoff (1983), a wealth of studies has aimed to answer this question by comparing the out-of-sample predictive ability of economic exchange rate models to random walk forecasts, with mixed success (for an overview, see Rossi 2013). Engel and West (2005) show, however, that exchange rates may be unpredictable if the fundamentals follow a random walk. Engel and West (2006) and Engel et al. (2019) therefore examine the in-sample explanatory power of fundamentals using variables typically included in Taylor-type policy rules. In this paper, we examine whether taking into account slow-moving trends, such as changes in the inflation target or the natural rate of unemployment, improves the in-sample explanatory power of exchange rate models.

To motivate our empirical framework, we derive a partial equilibrium expression for the bilateral real exchange rate, assuming that each country's central bank targets short-term interest rates according to a Taylor rule with a time-varying inflation target and a time-varying natural rate of unemployment. Combining these Taylor rules with a no-arbitrage condition reveals that the current real exchange rate depends on future expected trend inflation, inflation gaps, unemployment gaps, and short-term interest rates. The model implies that the trend and gap components may have a different impact on the exchange rate. A decline in inflation is associated with an appreciation of the currency if the central bank lowers its inflation target. By contrast, a decline in inflation below target is associated with a depreciation, if the central bank cuts its policy interest rate more than one-for-one.

Such considerations imply that the researcher needs to estimate latent variables such as trend inflation and the inflation gap. In this paper, we propose estimating these quantities with an unobserved components stochastic volatility (UC-SV) model, similar to Stock and Watson (2007). Trend inflation and trend unemployment follow non-stationary processes with stochastic volatility. This choice is common in the recent literature estimating trend inflation over different monetary policy regimes (see Ascari and Sbordone 2014, and references therein). The stationary components are assumed to follow an AR(1) model with stochastic volatility. The multivariate unobserved components model is combined with an additional equation that links the real exchange rate to the latent components. This equation is closely related to existing models based on Taylor rule fundamentals but discriminates between gap and trend components.

This paper contributes to the existing literature along three relevant dimensions. The first contribution of our paper is to provide an integrated framework to model real exchange rates. We allow for simultaneous estimation of the exchange rate equation and the latent components included in the unemployment and inflation equations. The vast majority of existing literature applies a two-step estimation approach. This implies that some point estimate of the output gap is used as a plug-in estimate in the exchange rate equation (see e.g. Engel and West 2006, Molodtsova and Papell 2009, Byrne et al. 2016, Huber 2017). However, this neglects uncertainty surrounding these latent components. Our method, in contrast, provides more flexibility by explicitly taking into account any statistical variation in the latent gap and trend components while capturing salient features like heteroscedastic shocks in the measurement and state equations. To assess whether the distinction between trend inflation and the inflation gap is supported by the data, we propose a novel model specification prior that enables testing whether it is necessary to estimate separate coefficients related to gap and trend components.

Second, we take into account changes in the policy rule to explain exchange rate fluctuations. Huber (2017) and Hauzenberger and Huber (2018) show that changes in policy rules can explain exchange rate fluctuations in particular during the low inflation period and since the financial crisis. Most studies implicitly assume a constant inflation target (see e.g. Molodtsova and Papell 2009). Ilzetzki et al. (2019), however, suggest that the pronounced decline in exchange rate volatility since 1975 may stem from a trend decline in inflation. We interpret this as a decline in the implicit inflation target of the central bank. For the US, this is supported by Surico (2008) who shows that the Federal Reserves' implicit inflation bias has disappeared in the post-Volcker regime. Moreover, Martin and Milas (2004) estimate a structural model for the UK economy assuming that the inflation target declined over time.

Third, we contribute to recent research on the uncovered interest parity (UIP) puzzle. These studies aim to explain the low or even negative correlation between exchange rate changes and the interest rate differential by the fact that interest rates are driven by a variety of shocks. Engel et al. (2019) find that including inflation in an UIP regression renders the coefficient on the interest rate differential insignificant, while high inflation leads to a subsequent appreciation of the currency. They show in a theoretical model that including liquidity shocks, in addition to monetary policy shocks, can account for this pattern. In addition, Chinn and Meredith (2004), Chinn and Quayyum (2012) examine whether UIP holds at short- and long horizons. In line with our findings, they show that UIP holds better at long horizons, but, less so for Japan and Switzerland since the financial crisis. Chinn and Zhang (2018) use a New Keynesian DSGE model to show that conventional monetary policy rules, in combination with the effective lower bound, can explain this pattern. Our paper suggests that shocks to trend inflation and the inflation gap are another potential source of the UIP puzzle.

The empirical findings can be summarized as follows. Our proposed UC-SV model receives substantial data support as measured by the deviance information criterion. This strong performance can be explained through our model selection prior, with posterior restriction probabilities pointing towards the necessity to decompose US inflation in its trend and cyclical component while for the home country, including the level of inflation appears to be sufficient. In terms of describing salient features of the involved exchange rates, the UC-SV model captures the major up- and downturns of bilateral real exchange rates against the US Dollar for a panel comprising of six economies during the post-Bretton Woods era. In fact, the correlations between the model-based predictions and the actual real exchange rates are as high as 0.58. A benchmark model, which is estimated on the same information set but does not discriminate between trend and gap components, yields lower correlations comparable to existing studies (see Engel and West 2006, Mark 2009). In terms of reproducing long-run trends of nominal exchange rates, the UC-SV model is capable of capturing all major low frequency movements over the last 40 years. In particular, we show that the model works particularly well before 1990, when trend inflation was volatile in many countries in our sample. Finally, the model successfully mimics the actual exchange rates with respect to several key time series properties. More specifically, we accurately reproduce the persistence of the real exchange rates and the correlations with other macroeconomic variables. Especially in terms of replicating the persistence of the real exchange rate, our approach sharply improves upon simpler benchmark specifications.

In what follows, Section 2 motivates the UC-SV model by deriving a partial equilibrium expression for the real exchange rate in terms of future expected fundamentals. Then, Section 3 outlines the empirical strategy adopted along with the corresponding prior specification. Finally, Section 4 presents the empirical results while the last section summarizes and concludes the paper.

2 Theoretical framework

Following Engel and West (2006), we derive an expression for the real exchange rate in terms of future expected fundamentals if monetary policy in two countries is characterized by Taylor rules. All equations are shown in log-linearized terms. Let the short-term policy interest rate i_t in the home economy be determined as

$$i_t = i_{t-1} + \gamma_\pi E_t \hat{\pi}_{t+1} + \gamma_u E_t \hat{u}_{t+1} + \gamma_q q_t + \varepsilon_t. \tag{1}$$

The central bank in the home economy targets the short-term interest rate as a function of deviations of expected inflation from the target ($E_t\hat{\pi}_{t+1}$), deviations of the expected unemployment rate from its natural level ($E_t\hat{u}_{t+1}$) and of the lagged interest rate, whereas ε_t is a monetary policy innovation.[1] The inflation and unemployment gaps are defined as $\hat{\pi}_t = \pi_t - \bar{\pi}_t$ and $\hat{u}_t = u_t - \bar{u}_t$, respectively. Therefore, the inflation target ($\bar{\pi}_t$) as well as the natural rate of unemployment ($\bar{u}_t$) change over time. As is standard in the literature $\gamma_\pi > 0$, $\gamma_u < 0$ such that the central bank increases its policy interest rate in response to a higher inflation gap or a lower unemployment gap.

We follow Engel and West (2006) and assume that the home central bank responds to the real exchange rate defined as $q_t = e_t - p_t + p_t^*$. The nominal exchange rate (e_t) is expressed as the price of one unit of the foreign currency in terms of domestic currency such that a rise in the exchange rate implies a depreciation of the home currency. Furthermore, p_t and p_t^* denote the domestic and foreign price levels, respectively. We assume that $\gamma_q > 0$, implying that the central bank lowers the interest rate when the exchange rate

[1] This specification reflects studies that find movements in trend inflation over time (Ascari and Sbordone 2014), changes in the non-accelerating inflation rate of unemployment over time (Gordon 1998) and relevant interest rate smoothing behavior of central banks (Coibion and Gorodnichenko 2012).

appreciates in real terms.

We depart from Engel and West (2006) in three ways. First, we assume that the central bank responds to an unemployment gap rather than an output gap. This is mainly because our empirical analysis uses monthly data. Second, we assume that the central bank responds to the future expected unemployment gap rather than the current one. Therefore, we take into account many central banks publish forecasts and justify their interest rate decisions based on future expected developments.[2] Third, we assume that the central bank follows a Taylor rule with interest rate smoothing (see e.g. Coibion and Gorodnichenko 2012). According to Giannoni and Woodford (2003) interest rate smoothing is a feature of optimal simple interest rate rules.

The central bank in the foreign economy targets the short-term interest rate using an analogous rule, except that it does not respond to the real exchange rate:[3]

$$i_t^* = i_{t-1}^* + \gamma_\pi E_t \hat{\pi}_{t+1}^* + \gamma_u E_t \hat{u}_{t+1}^* + \varepsilon_t^*, \tag{2}$$

where all foreign variables are labeled by an asterisk.

Furthermore, we assume that an uncovered interest parity relationship holds period-by-period:[4]

$$i_t - i_t^* = E_t[\Delta q_{t+1} + \pi_{t+1} - \pi_{t+1}^*]. \tag{3}$$

Replacing the interest rate differential by the two policy rules and rearranging terms we obtain:

$$\begin{aligned} q_t &= \rho E_t q_{t+1} + (1-\gamma_\pi)\rho E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) + \rho E_t(\bar{\pi}_{t+1} - \bar{\pi}_{t+1}^*) \\ &\quad -\gamma_u \rho E_t(\hat{u}_{t+1} - \hat{u}_{t+1}^*) - \rho(i_{t-1} - i_{t-1}^*) - \rho(\varepsilon_t - \varepsilon_t^*). \end{aligned} \tag{4}$$

with $\rho = \frac{1}{1+\gamma_q}$. Solving the equation forward allows to express the real exchange rate in terms of future expected fundamentals. This expression is the present-value solution of

[2] Batini and Haldane (1999) provide simulation evidence that such rules indeed perform well in a theoretical open economy model. Giannoni and Woodford (2003), however, provide a more critical appraisal of this practice.

[3] The Taylor rule parameters in the home and foreign economy are homogeneous for ease of exposition. In the empirical application we relax this restriction.

[4] A risk premium term would be straightforward to incorporate (see Engel and West 2006).

Engel and West (2005).

$$
\begin{aligned}
q_t &= \rho^{J+1} E_t q_{t+J+1} + (1-\gamma_\pi) E_t \sum_{j=0}^{J} \rho^{j+1} (\hat{\pi}_{t+j+1} - \hat{\pi}^*_{t+j+1}) \\
&\quad + E_t \sum_{j=0}^{J} \rho^{j+1} (\bar{\pi}_{t+j+1} - \bar{\pi}^*_{t+j+1}) - E_t \sum_{j=0}^{J-1} \rho^{j+1} (i_{t+j} - i^*_{t+j}) \\
&\quad - \gamma_u E_t \sum_{j=0}^{J} \rho^{j+1} (\hat{u}_{t+j+1} - \hat{u}^*_{t+j+1}) - \rho (i_{t-1} - i^*_{t-1}) - \rho (\varepsilon_t - \varepsilon^*_t).
\end{aligned} \tag{5}
$$

Despite the partial equilibrium nature of the analysis some interesting insights emerge. Reflecting the findings by Engel and West (2005), this equation suggests that changes in trend inflation, if they occur, will dominate fluctuations of the real exchange rate. To see this, assume that the home inflation gap follows a stationary AR(1) process with autoregressive parameter φ and home trend inflation follows a random walk. For $J \to \infty$, it is straightforward to show that the partial equilibrium effect of a change in the inflation gap and trend inflation amount to

$$
\begin{aligned}
\frac{\partial q_t}{\partial \hat{\pi}_t} &= \frac{(1-\gamma_\pi)}{1-\rho\varphi}, \\
\frac{\partial q_t}{\partial \bar{\pi}_t} &= \frac{1}{1-\rho}.
\end{aligned} \tag{6}
$$

Analogous expressions with the opposite sign hold for changes in the foreign inflation gap and foreign trend inflation. It follows that changes in domestic or foreign trend inflation will have, if they occur, a larger effect on the exchange rate in absolute value.[5]

In addition, changes in trend inflation and the inflation gap affect the exchange rate differently. Because $1-\gamma_\pi < 0$, an decline in the inflation gap causes a depreciation because the central bank lowers the policy rate by more than one for one.[6] By contrast, a decline in trend inflation causes an appreciation because the central bank does not change

[5] For example, for $\rho = 0.9,\ \varphi = 0.7,\ \gamma_\pi = 1.5$, the derivative amounts to -1.4 for the inflation gap and to 10 for trend inflation.

[6] This is in line with Engel et al. (2008) showing that, if the Taylor principle holds in a Taylor rule without interest rate smoothing, an increase in the expected inflation gap at home relative to the foreign economy implies a real appreciation.

the policy rate. Similar to Engel et al. (2019), the source of the shock causing the change in inflation matters whether we will observe an appreciation or depreciation.

To map the theoretical equation to empirical data, we need to form expectations about nominal short-term rates, the inflation and unemployment gaps, as well as future trend inflation. In what follows we outline the empirical strategy to model the decomposition and the future expected evolution of these measures.

3 Empirical framework

We propose a multivariate unobserved components stochastic volatility (UC-SV) model to describe the dynamics of the fundamentals. The model may be viewed as an open economy variant of earlier UC-SV specifications that aim to model inflation and unemployment dynamics by decomposing the respective variables in non-stationary trend and stationary gap components (for similar modeling approaches, see e.g., Gordon 1998, Stock and Watson 2007, Stella and Stock 2015, Chan et al. 2016, 2018, Hwu and Kim 2019).

3.1 The unobserved components stochastic volatility model

Let us store the observed inflation and unemployment series measured at time $t = 1, \ldots, T$ in a 4×1 vector $\boldsymbol{x}_t = (\pi_t, \pi_t^*, u_t, u_t^*)'$. We assume $\boldsymbol{x}_t$ may be decomposed as follows

$$\boldsymbol{x}_t = \overline{\boldsymbol{f}}_t + \hat{\boldsymbol{f}}_t + \boldsymbol{\varepsilon}_t, \tag{7}$$

$$\begin{pmatrix} \overline{\boldsymbol{f}}_t \\ \hat{\boldsymbol{f}}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Phi} \end{pmatrix} \begin{pmatrix} \overline{\boldsymbol{f}}_{t-1} \\ \hat{\boldsymbol{f}}_{t-1} \end{pmatrix} + \begin{pmatrix} \overline{\boldsymbol{\eta}}_t \\ \hat{\boldsymbol{\eta}}_t \end{pmatrix}, \tag{8}$$

with $\overline{\boldsymbol{f}}_t = [\overline{\pi}_t, \overline{\pi}_t^*, \overline{u}_t, \overline{u}_t^*]'$ being a 4×1 vector of latent trend components of inflation and unemployment at home and abroad. Similarly, $\hat{\boldsymbol{f}}_t = [\hat{\pi}_t, \hat{\pi}_t^*, \hat{u}_t, \hat{u}_t^*]'$ denotes a 4×1 vector of (stationary) latent gap components of inflation and unemployment. We assume that $\boldsymbol{\Phi} = \text{diag}(\phi_\pi, \phi_\pi^*, \phi_u, \phi_u^*)$ is a 4×4-dimensional matrix of autoregressive coefficients with absolute value below unity. This ensures that $\hat{\boldsymbol{f}}_t$ is mean reverting and thus permits us to interpret $\hat{\boldsymbol{f}}_t$ as a vector containing the inflation and unemployment gap, respectively.[7]

Finally, $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\zeta}_t = [\overline{\boldsymbol{\eta}}_t', \hat{\boldsymbol{\eta}}_t']'$ are normally distributed vector white noise errors with time-varying variance covariance matrices $\boldsymbol{\Sigma}_t$ and $\boldsymbol{V}_t$. We assume that $\boldsymbol{\Sigma}_t$ is a diagonal matrix with typical element σ_{jt}^2 $(j = 1, \dots, 4)$ and $\boldsymbol{V}_t$ is a full matrix that can be decomposed as

$$\boldsymbol{V}_t = \boldsymbol{A}\boldsymbol{S}_t\boldsymbol{A}', \tag{9}$$

where $\boldsymbol{A}$ is a 8×8-dimensional lower triangular matrix with unit diagonal and typical non-zero off-diagonal element a_j and $\boldsymbol{S}_t = \text{diag}(s_{1t}, \dots, s_{8t})$ contains the stochastic volatilities of the latent factors on its main diagonal.

We complete the description of our empirical model by stacking the logarithm of the volatilities in $\boldsymbol{\Sigma}_t$ and $\boldsymbol{S}_t$ in a generic vector $\boldsymbol{h}_t$, with typical element denoted by h_{it}. Following Kastner and Frühwirth-Schnatter (2014), we assume that each h_{it} evolves according to

$$h_{it} = \mu_i + \rho_i(h_{it-1} - \mu_i) + \sqrt{\vartheta_i}v_{it}, \tag{10}$$

where μ_i is the level of the log-volatility, $\rho_i \in (-1, 1)$ denotes the autoregressive parameter and ϑ_i denotes the variance of the log-volatility. This choice ensures that the volatility is bounded in the limit and rules out odd behavior related to random walk state equations for log-volatilities.

The UC-SV model explicitly discriminates between components that are non-stationary, capturing trends in the respective macroeconomic variable, and stationary processes that capture the high frequency behavior. To improve the fit of the model we moreover assume that all components are allowed to follow distinct stochastic volatility processes.

The specification described by Eqs. (7) to (8) is loosely related to the model put forward by Stella and Stock (2015). The key difference is that the multivariate unobserved component model of Stella and Stock (2015) assumes that, while both unemployment and inflation feature a non-stationary trend term, there exists only a single common stationary factor that can be interpreted as the unemployment gap. This common factor enters the

[7] Notice that, strictly speaking, the inflation gap is defined as the deviation of inflation from trend inflation. In our framework, that would imply that the inflation gap equals the first two elements of $\hat{\boldsymbol{f}}_t + \boldsymbol{\varepsilon}_t$. However, and since the measurement error variances of $\boldsymbol{\varepsilon}_t$ are typically close to zero, we refer to $\hat{\boldsymbol{f}}_t$ as the gap component. All empirical results reported in Section 4 remain qualitatively the same if we set $\boldsymbol{\varepsilon}_t = \boldsymbol{0}$ in Eq. (7).

inflation equation as an additional covariate, implying that the stationary part of inflation is driven by the unemployment gap plus a white noise shock. Our approach is slightly more flexible. This increased flexibility is necessary since we found that, for non-US data, using a single gap component is not sufficient to explain medium frequency movements in inflation and the corresponding factor loading close to zero. Moreover, another distinction between our proposed model and this specification is that we model unemployment and inflation across two countries whereas the specification of Stella and Stock (2015) is estimated on US data only.

3.2 Relation to the real exchange rate

We can derive an approximation that maps the empirical model described in subsection 3.1 to the theoretical exchange rate model. If we assume that the discount factor ρ is close to unity and assume that the expectations hypothesis holds, $E_t \sum_{j=0}^{J-1}(i_{t+j} - i^*_{t+j})$ is approximately J times the interest rate differential for J-period bonds which we denote as $J(b_{J,t} - b^*_{J,t})$.[8] Furthermore, for a discount factor close but below unity we have for large J that $E_t \rho^{J+1} q_{t+J+1} \approx 0$. Finally, under the structure of the UC-SV model we have that expectations of the gap components are formed as $E_t \hat{\pi}_{t+j} = \phi_\pi^j \hat{\pi}_t, E_t \hat{\pi}^*_{t+j} = \phi_\pi^{*j} \hat{\pi}^*_t$, $E_t \hat{u}_{t+j} = \phi_u^j \hat{u}_t$ and $E_t \hat{u}^*_{t+j} = \phi_u^{*j} \hat{u}^*_t$. Since the trend components follow a random walk process the expectations are given by $E_t \bar{\pi}_{t+j} = \bar{\pi}_t, E_t \bar{\pi}^*_{t+j} = \bar{\pi}^*_t, E_t \bar{u}_{t+j} = \bar{u}_t$ and $E_t \bar{u}^*_{t+j} = \bar{u}^*_t$. For large J and a discount factor close to unity we can approximate the exchange rate relationship in Eq. (5) as:

$$\begin{aligned} q_t \quad \approx \quad & \frac{1-\gamma_\pi}{1-\phi_\pi}(\hat{\pi}_t - \hat{\pi}^*_t) - \frac{\gamma_u}{1-\phi_u}(\hat{u}_t - \hat{u}^*_t) \\ & +(J+1)(\bar{\pi}_t - \bar{\pi}^*_t) - J(b_{J,t} - b^*_{J,t}) - (i_{t-1} - i^*_{t-1}) - (\varepsilon_t - \varepsilon^*_t). \end{aligned} \tag{11}$$

The terms involving the gap components are exact for $J \to \infty$ and $\rho \to 1$. However, it

[8] Under the expectations hypothesis, and abstracting from transaction costs, a no-arbitrage condition requires that the expected return of rolling over one-period zero-coupon bonds J times equals the return of holding a J-period zero-coupon bond until it matures. Formally, we have that $(1 + b_{J,t})^J = \prod_{j=0}^{J-1}(1 + E_t i_{t+j})$. Using a log-linear approximation around zero, that is $\ln(1 + x) \approx x$ for small x, yields $J b_{J,t} = \sum_{j=0}^{J-1} E_t i_{t+j}$.

is worth noting that these approximating assumptions are accurate even for finite J and relatively persistent processes.[9] In the empirical specification, we relax the assumption of parameter homogeneity across both countries' Taylor rules. The empirical model that relates the system described in the previous subsection to Eq. (11) is therefore given by

$$q_t = \boldsymbol{X}_t \boldsymbol{\beta} + \nu_t, \tag{12}$$

with

$$\begin{aligned} \boldsymbol{X}_t &= [1, i_{t-1}, i^*_{t-1}, \frac{\hat{\pi}_t}{1-\phi_\pi}, \frac{\hat{\pi}^*_t}{1-\phi^*_\pi}, \frac{\hat{u}_t}{1-\phi_u}, \frac{\hat{u}^*_t}{1-\phi^*_u}, \\ &\quad (J+1)\bar{\pi}_t, (J+1)\bar{\pi}^*_t, Jb_{Jt}, Jb^*_{Jt}], \end{aligned}$$

and $\nu_t \sim \mathcal{N}(0, \sigma^2_\nu)$ being a homoscedastic white noise error term. While it would be straightforward to allow for stochastic volatility in Eq. (12) we leave this possibility aside because we are mainly interested in capturing the dynamics of the exchange rate related to the first moment of the corresponding predictive density.

3.3 A model specification prior

To assess whether we have to discriminate between gap and trend components in Eq. (12), we follow a Bayesian approach and develop a model selection prior that allows for testing restrictions such as

$$\beta_{\hat{\pi}} = \beta_{\bar{\pi}}.$$

Here, we let $\beta_{\hat{\pi}} = \frac{\beta_4}{1-\phi_\pi}$ and $\beta_{\bar{\pi}} = (J+1)\beta_8$ denote transformed coefficients, implying that if the restriction holds, we obtain

$$\beta_{\hat{\pi}}(\hat{\pi}_t + \bar{\pi}_t) \approx \beta_{\hat{\pi}} \pi_t. \tag{13}$$

Notice that this is only an approximation since we introduce measurement errors to the observation equation. However, as mentioned in Footnote 6, the corresponding

[9] For an AR(1) process with autoregressive parameter $\rho = 0.97$ and forecast horizon $J = 120$, implying that $b_{J,t} - b^*_{J,t}$ is the difference in a ten-year government bond yield, the approximation error for the gap components amounts to 2.6% in terms of the correct finite-horizon expectation.

error variances are close to zero, implying that the state innovations are also small in magnitude. From a practical perspective, these restrictions effectively imply that we do not discriminate between gap and trend components of inflation in the home country. In the following, we test these restrictions not only for inflation in the home country but also for the foreign economy.

In what follows, we discuss the specific prior setup for the home country exclusively. The corresponding prior distributions on foreign quantities are similar and indicated by an asterisk. The prior distribution on $\beta_{\hat{\pi}}$ reads

$$\beta_{\hat{\pi}} \sim \mathcal{N}(\beta_{\bar{\pi}}, \tau_{\pi 0})\delta_{\pi} + \mathcal{N}(0, \tau_{\pi 1})(1 - \delta_{\pi}), \tag{14}$$

with δ_{π} being an indicator with $\text{Prob}(\delta_{\pi} = 1) = 1/2$ and $\tau_{\pi 0} \ll \tau_{\pi 1}$ denoting prior scaling parameters with $\tau_{\pi 0}$ set close to zero. This prior essentially implies that if $\delta_{\pi} = 1$, we strongly push the corresponding posterior estimate towards parameter homogeneity, implying that we do not discriminate between trend inflation and the inflation gap. This is commonly referred to as the 'spike' component. By contrast, if $\delta_{\pi} = 0$ we introduce only little prior information and allow for differences across parameters. This is the 'slab' component of the Gaussian prior.

Equation (14) can be rewritten as follows:

$$\beta_4 \sim \mathcal{N}\left(\beta_{\tilde{\pi}}(1 - \phi_{\pi}), (1 - \phi_{\pi})^2 \tau_{\pi 0}\right)\delta_{\pi} + \mathcal{N}\left(0, (1 - \phi_{\pi})^2 \tau_{\pi 1}\right)(1 - \delta_{\pi}).$$

This hierarchical prior structure is closely related to stochastic search variable selection priors (SSVS, George and McCulloch 1993) that allow for flexible testing whether it is necessary to discriminate between gap and trend components in our regression model. The main difference to standard SSVS priors is that the spike component is centered on $\beta_{\tilde{\pi}}(1 - \phi_{\pi})$. A similar specification has been used by Koop and Korobilis (2016) in panel VAR models to select cross-country restrictions within a Bayesian framework .

The state space model described in the previous subsection and Eq. (12) are estimated using Bayesian methods. The prior setup related to the remaining quantities of the model and the Markov chain Monte Carlo (MCMC) algorithm is described in Appendix A.

4 Empirical findings

We estimate the model for the US Dollar against the currencies of a panel of six economies: Germany, UK, Japan, Canada, Sweden and Switzerland (see Appendix B for a detailed description of the data). For the DEM/USD exchange rate, the series is linked with the EUR/USD exchange rate after the introduction of the Euro. The real exchange rate is calculated using the same consumer price indices that are used in the estimation for the trend inflation rate. We use 10-year government bond yields to approximate the sum of future expected short-term interest rates and thus set $J = 120$ months. As short-term interest rates we use 3-month interbank or T-Bill rates. Finally, we use civilian unemployment rates to estimate the unemployment gaps.

4.1 Model selection criteria and posterior restriction probabilities

Before discussing selected in-sample features of our proposed modeling approach, it is worth illustrating that our model receives substantial data support. To this end, we consider the deviance information criterion (DIC, see Spiegelhalter et al. 2002) that is a Bayesian information criterion similar to the Akaike Information Criterion. The DIC is computed as follows

$$DIC = -4E_{\Xi}\left[\log p(\boldsymbol{q}|\Xi)\right] + 2\log p(\boldsymbol{q}|\overline{\Xi})], \tag{15}$$

whereby $\boldsymbol{q} = (q_1, \ldots, q_T)'$, Ξ denotes generic notation that stacks all unknowns of the model and $\overline{\Xi}$ denotes the posterior mean of Ξ.

The first term in Eq. (15) rewards in-sample fit while the second term penalizes model complexity and serves to measure the effective number of parameters. Since the set of models we propose allows for discriminating between gap and cycle components, considering only the model fit would ignore increased model complexity and favor more complex specifications. This issue is circumvented by using the DIC. Notice that the DIC is a negatively oriented criterion and the model with the lowest DIC value is preferred. The reason we adopt this measure as opposed to more traditional model selection criteria such as the Bayesian information criterion or the Akaike information criterion is that our model is a hierarchical latent variable model and computing standard information criteria is not straightforward. Specifically, it is not clear how to count the number of parameters and, in addition, what point estimators for the latent quantities in the model should be used to compute the likelihood.

Our benchmark model is a simple exchange rate regression that does not discriminate between gap and trend components. More specifically, this implies that $\boldsymbol{X}_t$ is replaced by $\tilde{\boldsymbol{X}}_t$ given by

$$\tilde{\boldsymbol{X}}_t = [1, i_{t-1}, i^*_{t-1}, \pi_t, \pi^*_t, u_t, u^*_t, b_{Jt}, b^*_{Jt}]. \tag{16}$$

The corresponding set of regression coefficients features a SSVS prior in the spirit of George and McCulloch (1993). The prior on the error variances is of inverted Gamma form with loosely informative hyperparameters.

To decide on on how relevant different components of our model specification are, we compare not only the benchmark and the UC-SV model but also assess whether the introduction of stochastic volatility pays off. Table 1 shows the DICs across different models and exchange rates, with lower values of the DIC signaling a better model performance. Two major findings emerge. First, comparing the homoscedastic to the heteroscedastic specifications indicates that allowing for stochastic volatility generally yields lower DICs. This finding is closely related to recent findings in the literature on macroeconomic forecasting that highlights the relevance of controlling for heteroscedasticity for forecasting macroeconomic quantities (see, for instance, Clark 2011, Clark and Ravazzolo 2015). Second, by considering the first two columns of Table 1, we observe that applying a gap-cycle decomposition translates into lower DIC values across all currency pairs considered. Thus, the increased complexity is outweighed by a better model fit.

After providing some evidence that our proposed model fits the data well, we now turn to analyzing whether our stochastic model selection prior effectively pushes the model towards the benchmark specification. The corresponding probabilities that the parametric restrictions hold are shown in Table 2. The first column refers to the posterior probability that $\delta_\pi = 1$ while the second column shows the posterior probability that $\delta^*_\pi = 1$. In this discussion, this refers to the probability that the restrictions are introduced for US-based

Table 1 — Deviance information criterion across models

	Heteroscedastic		Homoscedastic	
	UC-SV	Benchmark	UC-SV	Benchmark
DE	9965.71	10074.90	10033.46	10073.21
UK	9448.99	9529.39	9451.95	9527.29
JP	10535.74	10577.51	10517.90	10576.62
CA	9173.12	9219.57	9210.09	9220.40
SE	10184.45	10338.18	10192.77	10337.45
CH	10085.25	10158.67	10105.98	10157.75

Notes: This table reports the deviance information criterion (DIC) across models. UC-SV refers to the proposed multivariate unobserved components model, benchmark is the benchmark model that does not discriminate between gap and trend components. 'Homoscedastic' refers to models without stochastic volatility while 'Heteroscedastic' introduces SV in both, the measurement and state equations.

quantities.

Table 2 — Posterior probabilities of parameter homogeneity across exchange rates

	π_t	π_t^*
DE	0.44	0.39
UK	0.61	0.33
JP	0.70	0.53
CA	0.64	0.48
SE	0.68	0.15
CH	0.54	0.27

Notes: This table shows the posterior mean of the restriction indicators δ_π and δ_π^*. The first column presents the posterior probability that $\delta_\pi = 1$ while the second column shows the posterior probability that $\delta_\pi^* = 1$.

Table 2 reveals that for most exchange rate pairs, the posterior probabilities of parameter homogeneity associated with domestic quantities exceed 60 percent. Only for the DM/USD exchange rate we find a restriction probability that is smaller than 50 percent. This finding, however, does not carry over to US-based gap and cycle components. For US fundamentals, we observe that posterior restriction probabilities are smaller than 50 percent in all cases except for the USD/JPY exchange rate. The main take away from this discussion is that for domestic quantities, it is sufficient to include the inflation rate without discriminating between the inflation gap and trend. For US-based quantities, it seemingly pays off to estimate separate coefficients associated with gap and trend components. At this point, it is worth stressing that estimating a model that does not discriminate between domestic trend inflation and the inflation gap while allowing for this feature for US measures differs from our approach. Our Bayesian model selection prior implies that in a certain fraction of cases, we differentiate between the inflation trend and gap whereas in other cases, we do not. This implies that the resulting posterior mean estimates can be viewed as a weighted average between different model specifications. This shows some resemblance to Bayesian model averaging strategies that aim to explore a vast model space to identify promising models.

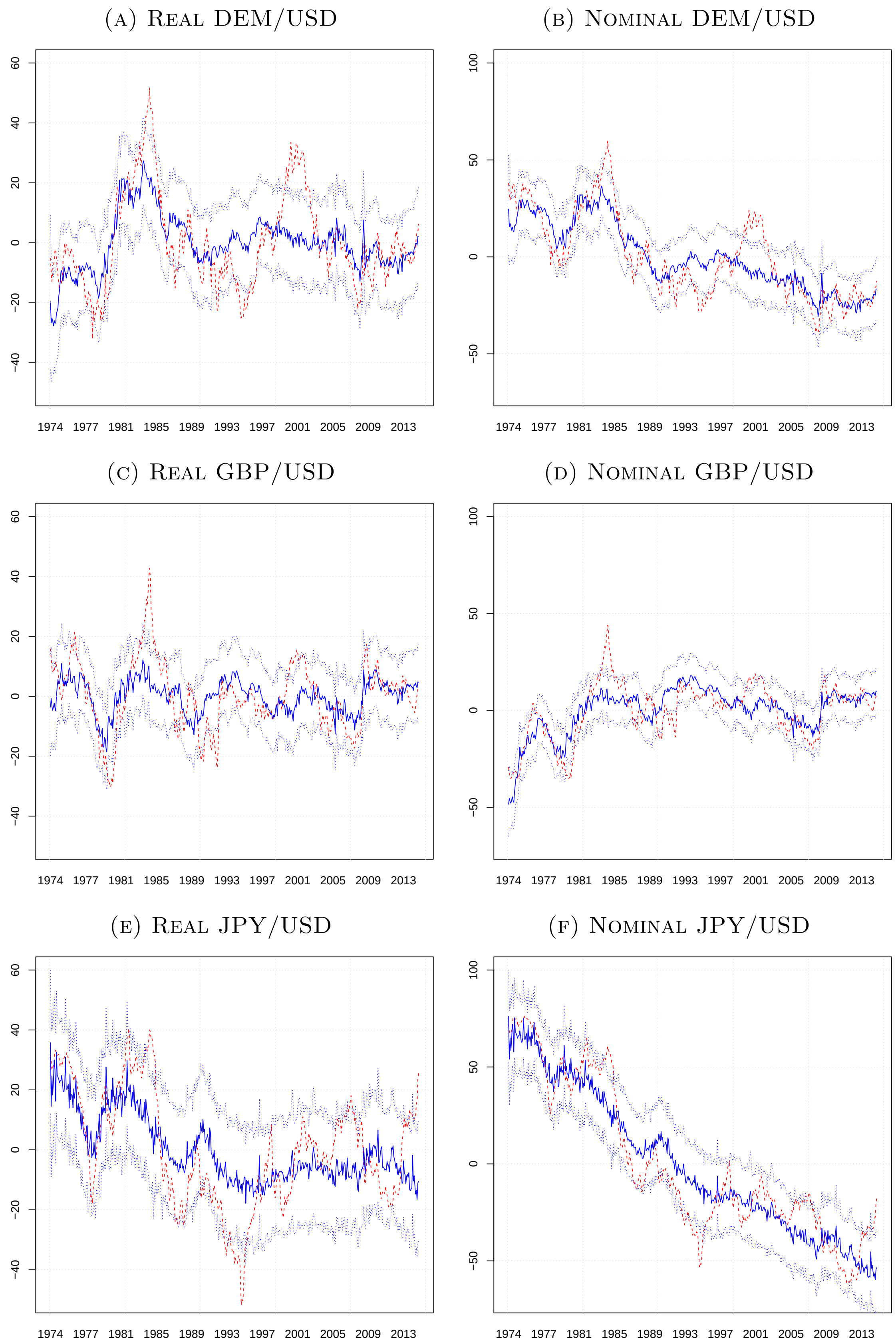
4.2 In-sample correlation

We next discuss the in-sample fit of the model in more detail. Figures 1 and 2 show the actual real and nominal exchange rates along with the mean and the 5th and 95th percentiles of the posterior distribution from the UC-SV model. The posterior distribution reflects uncertainty associated with the estimation of the UC-SV model as well as due to the estimation of the linear relationship of the exchange rate equation. For all countries, the posterior mean tracks major exchange rate movements well. The majority of turning points of the real exchange rate are captured by our model. Moreover, we match the appreciation trends of the nominal exchange rate, in particular, for Japan and Switzerland well.

In what follows, we discuss the episodes when the actual exchange rate moves outside of the 5th and 95th percentiles. In the mid-1980s, the real exchange rate leaves the credible bands for all countries except Canada. Similar problems of matching the strong US Dollar during this period are reported by Engel and West (2006), where they note that this period has been frequently labelled a US Dollar "bubble". This is in line with the idea that the fundamentals included in the extended model do not explain the strong Dollar.

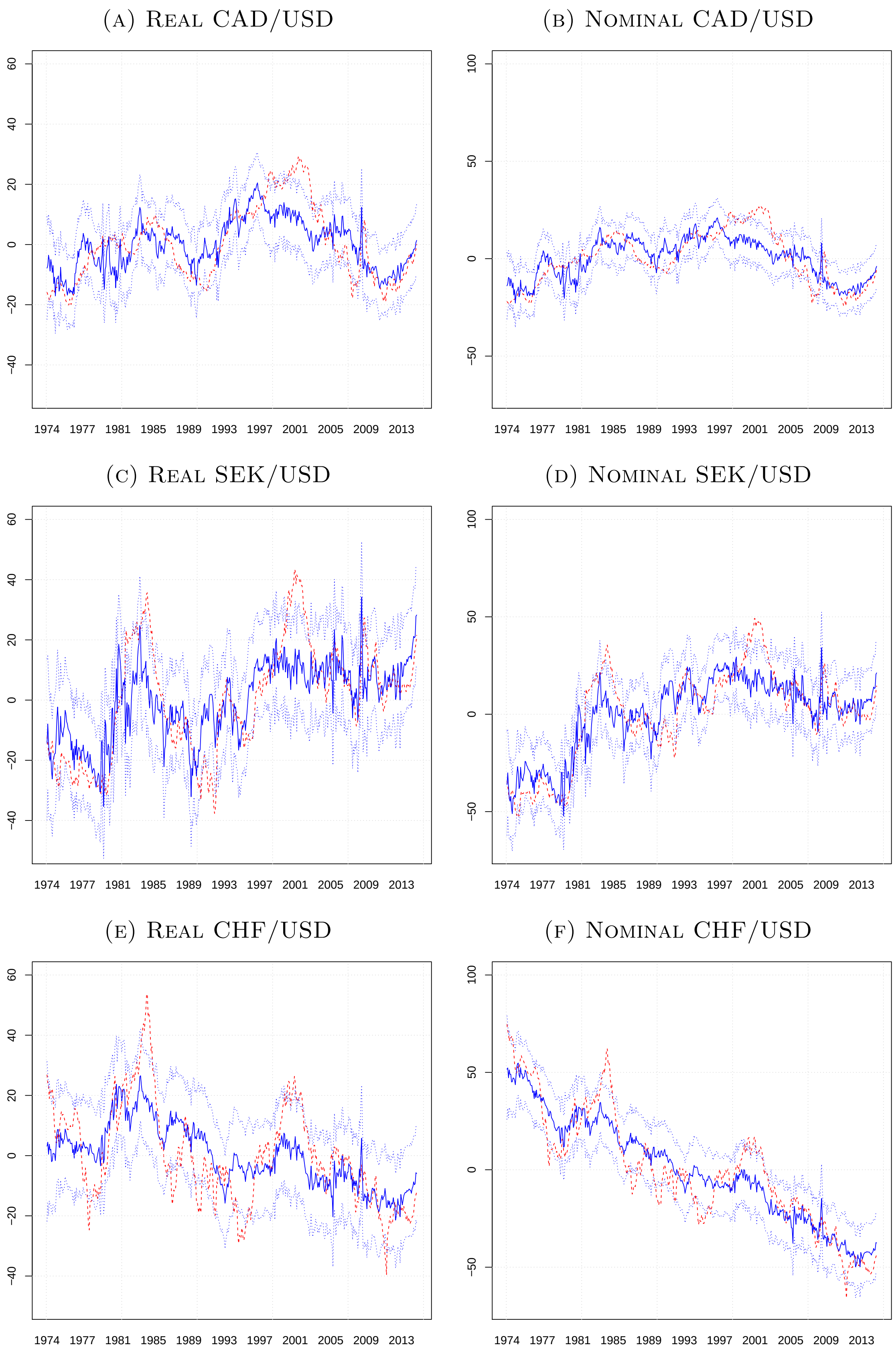
Starting in 1998, the US Dollar appreciated and rose outside of the 95th percentile for most currencies under consideration. We conjecture that this is closely related with several major economic crises that forced investors to reduce their non-USD exposure ("flight to safety"). More specifically, the Asian financial crisis, that hit the region between 1997 and 1998, was closely followed by the sovereign default of Russia and the

Figure 1 — Model predictions for large economies



Notes: Actual real and nominal US Dollar exchange rates are given by dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

Figure 2 — Model predictions for small economies



Notes: Actual real and nominal US Dollar exchange rates in dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

unwind of Long-Term Capital Management. Beside these developments in Asia, increased uncertainty surrounding the Argentinian crisis between 1998 and 2002 presumably contributed to the upward pressure on the US Dollar. Such save-haven considerations are probably not well captured in the factors affecting short-term interest rates via the Taylor rule.

Generally speaking, significant deviations from the model predictions occur when the Taylor rule is a poor approximation to monetary policy, for example, at the effective lower bound on short-term interest rates and during unconventional monetary policy actions. In 1978 and 2011, the real exchange rate leaves the credible bands for Switzerland when the short-term interest rate was constrained by the effective lower bound. Bäurle and Kaufmann (2018) argue that a currency is likely to appreciate strongly at the effective lower bound in response to modestly deflationary risk premium shocks because of increasing instead of declining real interest rates. In the late 1970s as well as in 2011, the SNB counteracted the appreciation by introducing a minimum exchange rate against the German Mark and the Euro, respectively. Also for Japan, we observe a substantial deviation from the prediction in 1995 when short-term interest rates fell to very low levels (to 0.4% in September 1995). These results are in line with Huber (2017) and Hauzenberger and Huber (2018) providing evidence for switching policy rules in exchange rate models when interest rates are constrained by the effective lower bound.

Similarly, the UC-SV approach may not fully include unconventional monetary policy actions and sharp and sudden changes in inflation expectations. For Japan, real and nominal exchange rates leave the percentiles in 2014. But the posterior mean moves into the opposite direction of the actual exchange rate already since 2012. This episode was governed by exceptional policy actions due to Abenomics which may not be appropriately reflected in the empirical UC-SV model: a higher inflation target, quantitative easing and an expansionary fiscal policy stance. These results are consistent with recent work on the UIP puzzle since the financial crisis (Chinn and Zhang 2018).

Another reason why our model fails may be that we ignore a risk premium that explains short-term fluctuations of exchange rates. To investigate this possibility we compute the correlation between the residual of the exchange rate models with various measures of uncertainty (stock market volatility and economic policy uncertainty). For brevity, we show these correlations in the online Appendix. For most exchange rates, we find significantly positive correlations between the residuals and the US VIX. The correlations range from 0.2 for the CHF/USD to 0.4 for the CAD/USD. This suggests that the exchange rates depreciate more than expected if stock market volatility in the US is high. We also find some significant correlations with non-US measures of uncertainty; but the correlations are smaller and less precisely estimated.

Table 3 — Correlation with actual exchange rate

		(A) Real		(B) Nominal	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.42	0.02	0.73	0.03
		$[0.35, 0.49]$	$[-0.08, 0.12]$	$[0.70, 0.79]$	$[-0.07, 0.13]$
	UC-SV	0.48	0.02	0.75	0.03
		$[0.41, 0.56]$	$[-0.05, 0.10]$	$[0.72, 0.79]$	$[-0.04, 0.10]$
GBP/USD	Benchmark	0.28	0.02	0.62	0.04
		$[0.20, 0.37]$	$[-0.06, 0.11]$	$[0.58, 0.69]$	$[-0.04, 0.12]$
	UC-SV	0.34	0.02	0.64	0.04
		$[0.26, 0.44]$	$[-0.03, 0.09]$	$[0.60, 0.69]$	$[-0.02, 0.10]$
JPY/USD	Benchmark	0.44	0.00	0.88	0.03
		$[0.37, 0.50]$	$[-0.08, 0.09]$	$[0.87, 0.92]$	$[-0.05, 0.11]$
	UC-SV	0.46	0.00	0.89	0.02
		$[0.35, 0.61]$	$[-0.06, 0.06]$	$[0.87, 0.92]$	$[-0.04, 0.08]$
CAD/USD	Benchmark	0.56	0.04	0.66	0.06
		$[0.51, 0.61]$	$[-0.05, 0.13]$	$[0.62, 0.75]$	$[-0.03, 0.14]$
	UC-SV	0.58	0.05	0.67	0.06
		$[0.48, 0.68]$	$[-0.02, 0.11]$	$[0.59, 0.75]$	$[-0.00, 0.12]$
SEK/USD	Benchmark	0.52	0.04	0.72	0.05
		$[0.46, 0.57]$	$[-0.03, 0.12]$	$[0.69, 0.80]$	$[-0.02, 0.13]$
	UC-SV	0.58	0.05	0.75	0.06
		$[0.51, 0.65]$	$[-0.01, 0.10]$	$[0.71, 0.80]$	$[0.00, 0.11]$
CHF/USD	Benchmark	0.47	0.03	0.86	0.04
		$[0.40, 0.53]$	$[-0.06, 0.12]$	$[0.84, 0.89]$	$[-0.05, 0.13]$
	UC-SV	0.50	0.04	0.87	0.04
		$[0.43, 0.59]$	$[-0.03, 0.10]$	$[0.85, 0.89]$	$[-0.02, 0.11]$

Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

Using the posterior distribution of the exchange rate prediction, we may investigate the model fit more formally by calculating the posterior distribution of the correlation with the actual exchange rate. The model predictions match the dynamics of the level of the exchange rate well, however, they do not explain exchange rate changes. Table 3 shows the posterior mean and percentiles for the correlation with the actual real and nominal exchange rates for each country. The first line is our benchmark model that does not control for the fact that trend inflation and trend unemployment may change over time. The second line gives the UC-SV model specification with the decomposition. Using the benchmark model we obtain correlations between 0.28 for the UK and 0.56 for Canada. The correlation for Germany at 0.42 is close to existing estimates by Engel and West (2006) and Mark (2009).

If we include trend inflation rates, the inflation gaps and the unemployment gaps separately, the correlation rises to 0.34 for the UK and to 0.58 for Canada. For Germany, the posterior mean correlation amounts to 0.48. The statistics broadly confirm our conclusions using model selection criteria. Quantitatively, however, the results are relatively modest. For the nominal exchange rate, the correlation is generally higher reflecting that we match the trends for Japan and Switzerland particularly well. But also, the correlation is substantial for Canada where the nominal exchange rate does not exhibit a strong secular trend.

For changes in exchange rates, the model does not outperform the benchmark. While the posterior mean correlation is usually higher for the UC-SV model when compared with the benchmark. In fact, the percentiles always include zero for both specifications. This suggests that we mainly capture the major exchange rate movements while month-to-month movements are not very well captured. This also supports Chinn and Meredith (2004) and Chinn and Quayyum (2012) who find that UIP holds better in the long- rather than in the short run.

If our model works well because of changes in trend inflation, we expect that the correlations differ mainly in the period before 1990.[10] Table 4 reports the correlations with the actual real exchange rate separately for the two time periods. Except for Canada, the correlation of the UC-SV prediction is higher before 1990 than afterward. In addition, except for Japan, the increase relative to the benchmark is higher before 1990. This

[10]For all countries, trend inflation varied more strongly before 1990 (see Figure 3 in the online Appendix).

Table 4 — Correlation with actual exchange rate before and after 1990

		(A) Real, before 1990		(B) Real, after 1990	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.62	−0.02	0.11	0.05
		[0.53, 0.69]	[−0.18, 0.15]	[−0.01, 0.23]	[−0.08, 0.18]
	UC-SV	0.68	−0.01	0.15	0.04
		[0.60, 0.76]	[−0.12, 0.12]	[0.03, 0.29]	[−0.05, 0.14]
GBP/USD	Benchmark	0.35	0.00	0.22	0.04
		[0.23, 0.47]	[−0.13, 0.13]	[0.10, 0.34]	[−0.08, 0.15]
	UC-SV	0.44	0.01	0.24	0.03
		[0.31, 0.56]	[−0.09, 0.11]	[0.14, 0.35]	[−0.05, 0.11]
JPY/USD	Benchmark	0.55	0.00	−0.02	0.01
		[0.44, 0.64]	[−0.13, 0.13]	[−0.14, 0.10]	[−0.10, 0.12]
	UC-SV	0.52	0.00	0.02	0.01
		[0.36, 0.67]	[−0.10, 0.09]	[−0.15, 0.30]	[−0.07, 0.08]
CAD/USD	Benchmark	0.36	0.02	0.59	0.05
		[0.23, 0.47]	[−0.14, 0.18]	[0.52, 0.65]	[−0.06, 0.16]
	UC-SV	0.41	0.04	0.61	0.05
		[0.22, 0.56]	[−0.08, 0.17]	[0.49, 0.71]	[−0.04, 0.13]
SEK/USD	Benchmark	0.47	−0.03	0.42	0.09
		[0.34, 0.57]	[−0.15, 0.09]	[0.32, 0.51]	[−0.01, 0.19]
	UC-SV	0.56	−0.02	0.46	0.08
		[0.46, 0.67]	[−0.11, 0.08]	[0.37, 0.56]	[0.01, 0.16]
CHF/USD	Benchmark	0.22	0.01	0.31	0.05
		[0.07, 0.36]	[−0.12, 0.14]	[0.19, 0.41]	[−0.07, 0.16]
	UC-SV	0.36	0.03	0.31	0.04
		[0.20, 0.51]	[−0.07, 0.13]	[0.17, 0.44]	[−0.04, 0.13]

Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

suggests that our model works well mainly because we capture changes in trend inflation.

An important aspect for an exchange rate model to match is the high persistence or near random walk properties of the real exchange rate. Table 5 shows the sample autocorrelation up to the third order for the actual real exchange rate along with the autocorrelation of the posterior means of the predictions. The benchmark model already implies a highly persistent real exchange rate. Nevertheless, the persistence of the exchange rate based on the benchmark model is lower than that of the actual real exchange rate for

Table 5 — Autocorrelation real exchange rate

		Log-level			Log-change		
		1st	2nd	3rd	1st	2nd	3rd
DEM/USD	Actual	0.98	0.96	0.93	0.01	0.04	0.04
	Benchmark	0.94	0.90	0.88	−0.14	−0.23	−0.03
	UC-SV	0.97	0.93	0.92	−0.08	−0.20	0.00
GBP/USD	Actual	0.97	0.94	0.90	0.33	0.02	0.05
	Benchmark	0.90	0.82	0.78	−0.13	−0.17	−0.05
	UC-SV	0.93	0.87	0.83	−0.09	−0.17	−0.05
JPY/USD	Actual	0.99	0.96	0.94	0.33	0.08	0.05
	Benchmark	0.90	0.89	0.87	−0.41	−0.03	−0.01
	UC-SV	0.93	0.92	0.90	−0.41	−0.03	−0.02
CAD/USD	Actual	0.99	0.98	0.97	0.21	0.05	0.03
	Benchmark	0.95	0.91	0.89	−0.15	−0.12	−0.10
	UC-SV	0.95	0.92	0.89	−0.13	−0.12	−0.10
SEK/USD	Actual	0.99	0.97	0.95	0.36	0.04	0.05
	Benchmark	0.86	0.76	0.73	−0.11	−0.26	−0.10
	UC-SV	0.91	0.84	0.81	−0.09	−0.25	−0.10
CHF/USD	Actual	0.98	0.96	0.93	0.27	0.03	0.02
	Benchmark	0.95	0.91	0.90	−0.17	−0.25	−0.04
	UC-SV	0.97	0.94	0.93	−0.13	−0.22	−0.05

Notes: Sample autocorrelation function for the actual real US Dollar exchange rate and sample autocorrelation function for the posterior mean of the model predictions up to 3rd order. The benchmark model does not take into account changes in the inflation and unemployment trends.

all countries and all lags. The UC-SV model is capable of explaining the higher persistence of the real exchange rate and matches the actual persistence more closely. The only where the model does not substantially improve over the benchmark is Canada.

Similar to the previous results, we do not make much progress matching the persistence of exchange rate changes. In the actual data, the first order autocorrelation is larger than zero for all countries. By contrast, the benchmark model implies a negative first order autocorrelation for exchange rate changes. Although the UC-SV model often yields a first order autocorrelation closer to zero, the improvement over the benchmark is modest.

As a further check whether the exchange rate model predicts the real exchange rate with reasonable properties, we compare correlations of the real exchange rate with the

Table 6 — Correlation of real exchange rate with fundamentals

		$\pi_t - \pi_t^*$	$u_t - u_t^*$	$i_t - i_t^*$	$b_t - b_t^*$
DEM/USD	Actual	0.15	0.19	−0.20	−0.44
	Benchmark	0.18	0.23	−0.24	−0.54
		[0.11, 0.25]	[0.16, 0.30]	[−0.31, −0.17]	[−0.59, −0.48]
	UC-SV	0.18	0.23	−0.24	−0.53
		[0.11, 0.25]	[0.15, 0.31]	[−0.30, −0.17]	[−0.58, −0.47]
GBP/USD	Actual	0.11	0.04	−0.06	−0.02
	Benchmark	0.14	0.04	−0.08	−0.02
		[0.05, 0.22]	[−0.04, 0.12]	[−0.16, 0.01]	[−0.10, 0.06]
	UC-SV	0.14	0.03	−0.07	−0.02
		[0.06, 0.21]	[−0.07, 0.13]	[−0.15, 0.00]	[−0.10, 0.05]
JPY/USD	Actual	0.17	−0.39	−0.07	0.20
	Benchmark	0.21	−0.47	−0.09	0.25
		[0.14, 0.28]	[−0.53, −0.41]	[−0.16, −0.02]	[0.17, 0.31]
	UC-SV	0.20	−0.50	−0.09	0.24
		[0.13, 0.27]	[−0.62, −0.38]	[−0.16, −0.02]	[0.17, 0.31]
CAD/USD	Actual	−0.03	0.54	−0.38	−0.18
	Benchmark	−0.04	0.63	−0.44	−0.21
		[−0.10, 0.03]	[0.58, 0.67]	[−0.50, −0.39]	[−0.27, −0.15]
	UC-SV	−0.03	0.59	−0.44	−0.21
		[−0.09, 0.03]	[0.47, 0.68]	[−0.50, −0.38]	[−0.27, −0.15]
SEK/USD	Actual	−0.06	0.38	−0.07	−0.43
	Benchmark	−0.07	0.45	−0.08	−0.50
		[−0.13, −0.00]	[0.39, 0.51]	[−0.14, −0.01]	[−0.56, −0.45]
	UC-SV	−0.07	0.44	−0.08	−0.49
		[−0.13, −0.01]	[0.35, 0.51]	[−0.13, −0.02]	[−0.54, −0.44]
CHF/USD	Actual	0.08	−0.32	−0.30	−0.47
	Benchmark	0.10	−0.38	−0.36	−0.56
		[0.03, 0.17]	[−0.44, −0.32]	[−0.42, −0.30]	[−0.61, −0.50]
	UC-SV	0.10	−0.37	−0.35	−0.55
		[0.03, 0.16]	[−0.46, −0.28]	[−0.41, −0.29]	[−0.60, −0.50]

Notes: Correlations of the actual and predicted real US Dollar exchange rate with differences in the fundamentals. The 5th and 95th percentiles are given in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

fundamentals. Table 6 shows that the model closely matches the correlation between the actual exchange rate and the fundamentals. The posterior mean is close to the actual correlation and the 5th and 95th percentiles mostly include the actual value. The actual value of the correlation lies outside of the percentiles for inflation in Japan, Canada and Switzerland and long-term bonds in Germany and Switzerland. However, even for these correlations the sign of the posterior mean is consistent with the sign of the actual correlation. For the benchmark model, the correlations also tend to track the actual relationship between the exchange rate and its fundamentals rather well.

Finally, we performed robustness checks with respect to the model specification (the results are available in the online Appendix). Replacing the stochastic trends with an estimate from the Hodrick and Prescott (1997) filter yields similar correlations between the actual and predicted real exchange rate as in the UC-SV model.[11] Then, we limited the sample to the pre-Euro era for Germany. The main results are robust. We even find a somewhat higher correlation of our prediction and the actual real exchange rate of 0.7. Engel and West (2006) find for a similar sample a correlation between their prediction and the real DEM/USD of 0.3. Moreover, we also assess the impact of allowing for SV not only in terms of measures of model adequacy like the DIC but also how results change if we turn off heteroscedasticity. In that case, results change only quantitatively, with the vast majority of findings remaining valid.

5 Closing remarks

Recent research has documented that trend inflation changes over time. We add an international dimension to this line of research and highlight that changes in trend inflation explain important aspects of exchange rate dynamics. We develop a multivariate UC-SV model that is theoretically motivated by assuming that central banks follow Taylor rules, but the inflation target as well as the natural rate of unemployment may change over time.

The UC-SV model succeeds in capturing major up- and downturns of the real US Dollar exchange rate against the currencies of six economies. In fact, the correlations of the model predictions with the actual real exchange rates are higher than in existing studies. While a benchmark model performs comparatively well, the improvements obtained by explicitly discriminating between non-stationary trend and stationary gap components are significant for all currencies under consideration. Looking at nominal exchange rates reveals that we are able to accurately reproduce major exchange rate

[11] We follow Ravn and Uhlig (2002) and set the smoothing parameter to 129,600 for monthly data.

trends observed over the last 40 years. Finally, the model successfully captures several key time series characteristics commonly found for real exchange rates. More specifically, we accurately reproduce the persistence of the real exchange rate and its correlation with other macroeconomic variables. As one would expect, however, accounting for changes in trend inflation has a more important effect for the pre-1990 period.

Our discussion shows that, although the model explains a larger share of exchange rate fluctuations than previous studies, it fails during episodes when the Taylor rule is unlikely to be an accurate description of the central banks' conduct of monetary policy. Improving the model predictions by accounting for unconventional monetary policy actions and constraints on the operational targets of central banks might be a promising avenue for future research.

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Online Appendix

Appendix A Prior setup and posterior simulation

The approach to estimation and inference is Bayesian. We thus have to specify suitable prior distributions for all coefficients of the UC-SV model.

Point of departure is a normally distributed prior for the initial value of $\boldsymbol{f}_t = (\overline{\boldsymbol{f}}'_t, \hat{\boldsymbol{f}}'_t)'$,

$$\boldsymbol{f}_1 \sim \mathcal{N}(\mathbf{0}, \underline{\boldsymbol{V}}_f). \tag{17}$$

Here $\underline{\boldsymbol{V}}_f$ is a diagonal prior variance-covariance matrix where we set the diagonal elements equal to ten, implying that we are relatively uninformative about the specific value of the initial state of the system.

For the diagonal elements of $\boldsymbol{\Phi}$ we also impose a normally distributed prior. More specifically, we set

$$\phi_{ii} \sim \mathcal{N}(\underline{\phi}_{ii}, \underline{v}_{\phi ii}) \text{ for } i = 1, \ldots, 4, \tag{18}$$

with $\underline{\phi}_{ii}$ and $\underline{v}_{\phi_{ii}}$ denoting prior mean and variance, respectively. We center the prior means associated with the inflation gap to 0.75 and the corresponding prior variance to $(0.1)^3$.[12] In addition, we set the prior mean related to the unemployment gap to 0.99, with prior variance set equal to $(0.1)^3$. This tight prior implies that the inflation gap is less persistent than the unemployment gap. A prior setup that is relatively uninformative on the autoregressive coefficients of the gap components yields results that are qualitatively similar. However, inspection of the posterior draws reveals that the likelihood is relatively uninformative on the persistence, and we thus experimented with different values of the parameters for the US to match the results presented in Stella and Stock (2015).

We use a Gaussian prior for the free elements of $\boldsymbol{A}_t$,

$$a_j \sim \mathcal{N}(\underline{a}_j, \underline{v}_{a_j}) \tag{19}$$

where we set $\underline{a}_j$ equal to zero and $\underline{v}_{a_j}$ equal to $(0.1)^3$. Again, this prior specification

[12] This is broadly consistent with findings on the persistence of the inflation gap for the US before the Great Moderation (see Cogley and Sbordone 2008, Cogley et al. 2010).

places considerable mass on the prior view that the shocks to the state equations are uncorrelated. Being effectively uninformative about a_j yields similar results but at the cost that the MCMC algorithm mixes somewhat slower.

For the priors on the level of the log-volatility μ_i we impose a normal prior with mean $\underline{\mu}_i$ and variance $\underline{v}_{\mu i}$,

$$\mu_i \sim \mathcal{N}(\underline{\mu}_i, \underline{v}_{\mu_i}). \tag{20}$$

We set $\underline{\mu}_i = 0$ and $\underline{v}_{\mu_i} = 10^2$ for $i = 1, \dots, 9$ to render this prior effectively uninformative. In addition, we impose a Beta prior on the persistence parameter ρ_i

$$\frac{\rho_i + 1}{2} \sim \mathcal{B}(b_0, b_1), \tag{21}$$

where we set $b_0 = 25$ and $b_1 = 5$ for all i leading to a prior mean of 0.83 with prior standard deviation of 0.07, thus placing considerable prior mass on high persistence regions of ρ_i. Note that this choice proves to be quite influential in practice since the likelihood typically carries little information about the persistence of the log-volatility.

Following Kastner and Frühwirth-Schnatter (2014) we use a non-conjugate Gamma prior on the variance of the log-volatility,

$$\vartheta_i \sim \mathcal{G}(1/2, \frac{1}{2B_\vartheta}). \tag{22}$$

The hyperparameter B_ϑ controls the tightness of the prior. It is straightforward to show that this prior implies

$$\pm\sqrt{\vartheta_i} \sim \mathcal{N}(0, B_\vartheta). \tag{23}$$

In the empirical application we set B_ϑ equal to unity. After experimenting with different values of B_ϑ, the specific choice of this hyperparameter proves to be rather unimportant in the present application. This prior setup has been motivated in Frühwirth-Schnatter and Wagner (2010) and provides several convenient properties. For instance, the Gamma prior does not bound ϑ_i away from zero and thus induces more shrinkage as the typical conjugate inverted Gamma prior.

For the elements of $\boldsymbol{\beta}$, we use the prior discussed in Section 3.3. The hyperparameters are chosen as follows.[13] For the spike variance, we use $\tau_{\pi 0} = 0.1 \times \hat{\sigma}_\pi^2$, where $\hat{\sigma}_\pi^2$ denotes the variance of the OLS estimator related to β_4. The slab variance is specified to equal $\tau_{\pi 1} = 10 \times \hat{\sigma}_\pi^2$, effectively rendering this prior weakly informative (conditional on δ_π).

Finally, we use an inverted Gamma prior for σ_ν^2,

$$\sigma_\nu^2 \sim \mathcal{IG}(c_0, c_1), \tag{24}$$

where c_0 and c_1 are set equal to $(0.1)^3$, rendering this prior effectively non-influential.

The Markov chain Monte Carlo algorithm iterates between the following steps:

- Simulate the full history of $\boldsymbol{f}_t$, denoted as $\boldsymbol{f}^T = (\boldsymbol{f}_1, \dots, \boldsymbol{f}_T)'$ conditional on all other parameters and the data using the well-known algorithm developed by Carter and Kohn (1994) and Frühwirth-Schnatter (1994).
- The parameters of the log-volatility in Eq. (10) and the full history of log-volatilities $h_i^T = (h_{i1}, \dots, h_{iT})'$ are simulated by means of the algorithm provided in Kastner and Frühwirth-Schnatter (2014), which proves to be an efficient alternative to other popular algorithms.[14]
- The autoregressive parameters of the state equations in Eq. (8) are sampled through Gibbs steps from their conditional Gaussian posterior distributions. To ensure stationarity we impose the constraint that all draws have to be smaller than unity in absolute values.
- Similarly, given the conjugacy of the prior setup employed, $\boldsymbol{\beta}$ is simulated from a normal distribution with well-known posterior mean and variance.
- The prior indicators δ_π and δ_π^* are simulated from Bernoulli distributions with a posterior restriction probability that takes a well known form (see, e.g., George and McCulloch 1993).
- For the covariance parameters a_j we follow Cogley and Sargent (2005) and rewrite the reduced-form errors as a set of simple regression models with innovations that are standard normally distributed. The normal prior on each a_j then yields a well-known Gaussian posterior density with known moments that can be used to simulate a_j.

[13] Here we discuss the prior setup for domestic quantities only. For foreign quantities, we use the same hyperparameter values.

[14] This step is implemented using the `R` package stochvol (Kastner 2015a,b).

- Finally, σ_{ν}^2 is sampled with a Gibbs step by noting that the conditional posterior is of a well-known form, namely an inverted Gamma distribution.

In the empirical application we repeat this algorithm 30,000 times and discard the first 15,000 iterations as burn-ins. Moreover we impose the restriction that the variance of the unemployment gap at home and abroad equals to 0.3. Since allowing for stochastic volatility in the measurement error and the errors of the gap components separately typically leads to empirical problems, we fix the variance of $\hat{u}_t$ and $\hat{u}_t^*$. Again, setting the variance equal to 0.3 is predicated by calibrating the model to match the trend unemployment rate and unemployment gap estimated by previous studies for the US.

Appendix B Data

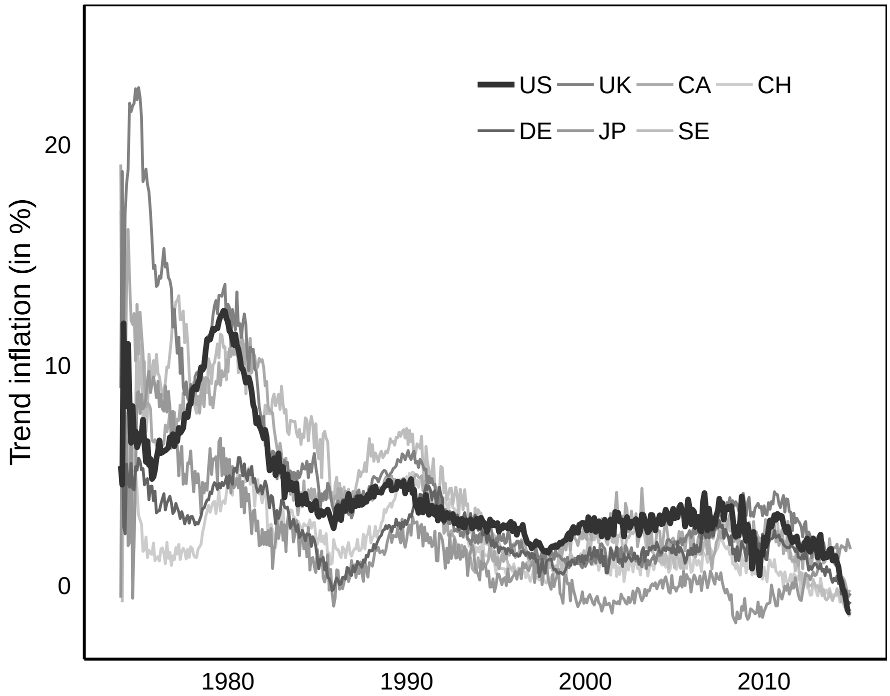
Table 7 — Data, sources, transformations

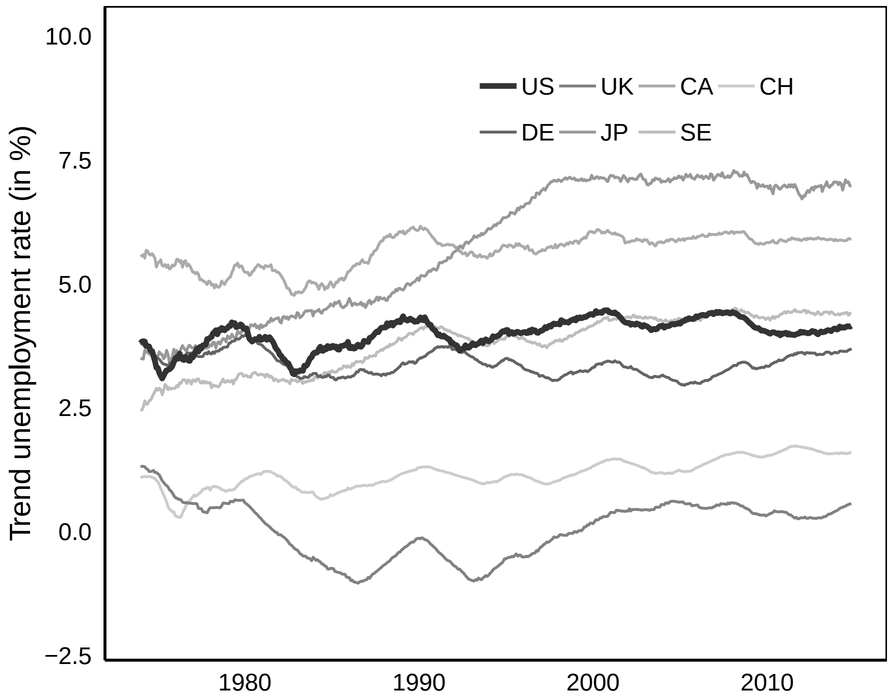
	Country	Identifier	Source	Comments
Exchange rates	CA	EXCAUS	FRB	
	JP	EXJPUS	FRB	
	SE	EXSDUS	FRB	
	CH	EXSZUS	FRB	
	UK	EXUSUK	FRB	Inverted
	DE	CCUSSP01DEM650N	MEI	Inverted, EUR/USD after euro changeover
CPI	CA	CANCPIALLMINMEI	MEI	Census X13 seas. adj.
	JP	JPNCPIALLMINMEI	MEI	Census X13 seas. adj.
	SE	SWECPIALLMINMEI	MEI	Census X13 seas. adj.
	CH	CHECPIALLMINMEI	MEI	Census X13 seas. adj.
	UK	GBRCPIALLMINMEI	MEI	Census X13 seas. adj.
	US	CPIAUCSL	BLS	
	DE	DEUCPIALLMINMEI	MEI	Census X13 seas. adj.
Unemployment rates	CA	LRUNTTTTCAM156S	MEI	
	JP	LRUN24TTJPM156N	MEI	Census X13 seas. adj.
	SE	LRHUTTTTSEM156S, SWEURHARMMDSMEI	MEI	Sources linked in 1983
	CH	LMUNRRTTCHM156N	MEI	Census X13 seas. adj.
	UK	LMUNRRTTGBM156S	MEI	
	US	UNRATE	BLS	
	DE		BA	Downloaded from Datastream
Short rates	CA	IR3TIB01CAM156N	MEI	Interbank rate
	JP	INTGSTJPM193N	IFS	T-Bill rate
	SE	IR3TIB01SEM156N	MEI	Linked with Riksbank data (see notes)
	CH	IR3TIB01CHM156N	MEI	Interbank rate
	UK	IR3TTS01GBM156N	MEI	T-Bill rate
	US	IR3TIB01USM156N	MEI	Interbank rate
	DE	IR3TIB01DEM156N	MEI	Interbank rate
Long rates	CA	IRLTLT01CAM156N	MEI	
	JP	INTGSBJPM193N	IFS	
	SE	IRLTLT01SEM156N	MEI	Linked with Riksbank data (see notes)
	CH	IRLTLT01CHM156N	MEI	
	UK	IRLTLT01GBM156N	MEI	
	US	IRLTLT01USM156N	MEI	
	DE	IRLTLT01DEM156N	MEI	
VIX	CA	TSX60	S&P	Start 10/2009
	JP			N/A
	SE			N/A
	CH	VSMI	SIX	Start 01/1999
	UK	.VFTSE	Datastream	Start 01/2000
	US	VIXCLS	FRED	Start 01/1990
	DE	.V1XI	Datastream	Start 12/1998
Policy uncertainty	CA		EPU	Start 01/1985
	JP		EPU	Start 01/1987
	SE		EPU	Start 01/1975
	CH		EPU	N/A
	UK		EPU	Start 01/1998
	US		EPU	Start 01/1985
	DE		EPU	Start 01/1993

Notes: All data, unless otherwise indicated, was retrieved from FRED, Federal Reserve Bank of St. Louis `https://research.stlouisfed.org/fred2/`. Data for short-term and long-term interest rates for Sweden was downloaded from `http://www.riksbank.se/en/The-Riksbank/Research/Historical-Monetary-Statistics-/Interest-and-stock-returns/`. Data for economic policy uncertainty indices were downloaded from `http://www.policyuncertainty.com/`.

Appendix C Additional results and robustness tests

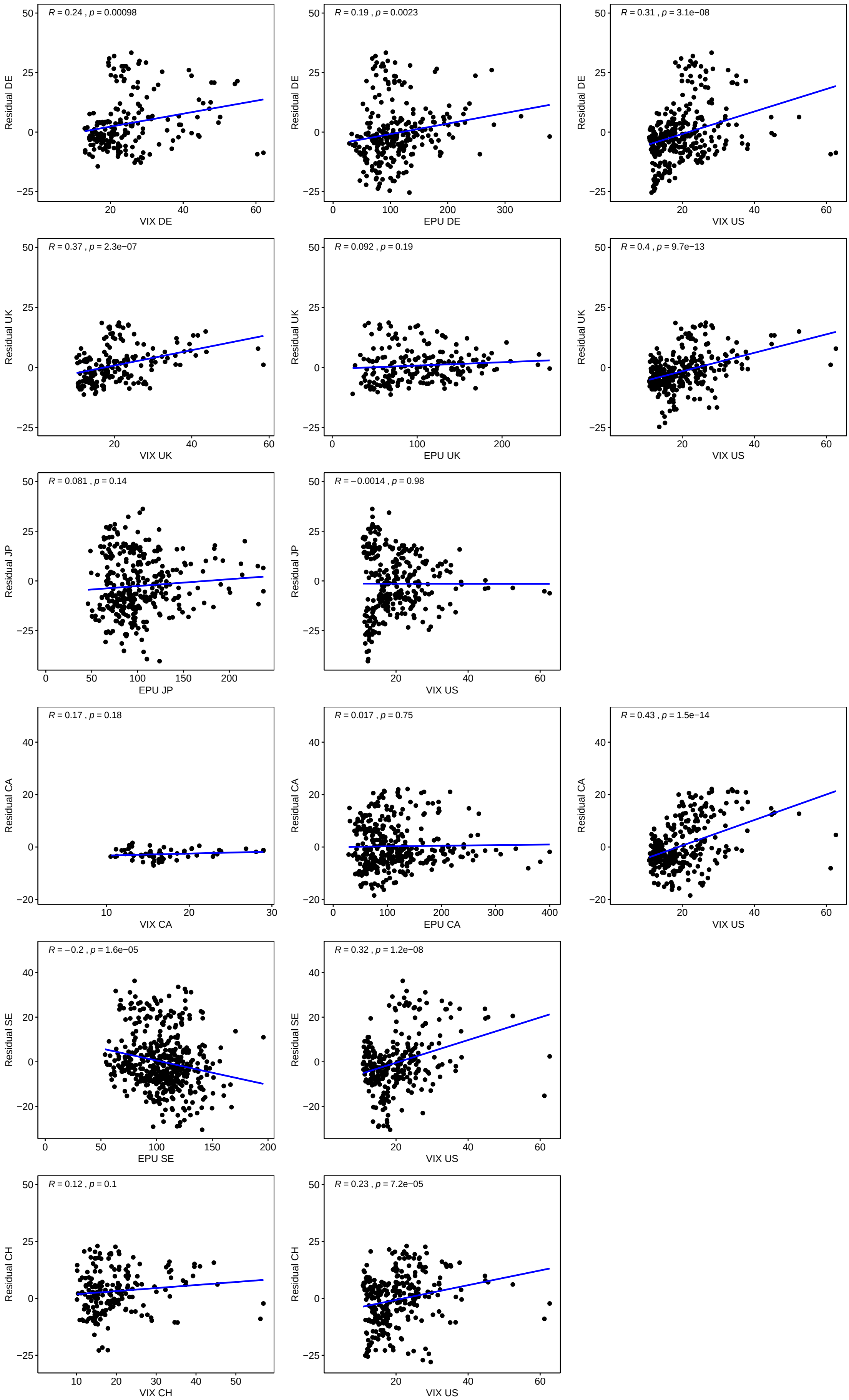
Figure 3 — Estimates of trends





Notes: Posterior mean of the estimated trend inflation rate (annualized) and trend unemployment rate. The results are based on 15,000 posterior draws.

Figure 4 — Correlation of residuals with uncertainty measures



Notes: Correlations of the residuals of the UC-SV model with various uncertainty measures (VIX and economic policy uncertainty; EPU). The sample range differs because most of the uncertainty measures start only in the 1990s or 2000s (see Appendix B).

Appendix C.1 Model without SV

TABLE 8 — CORRELATION WITH ACTUAL EXCHANGE RATE (WITHOUT SV)

		(A) Real		(B) Nominal	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.42	0.02	0.73	0.03
		[0.36, 0.49]	[−0.08, 0.12]	[0.70, 0.78]	[−0.08, 0.13]
	UC-SV	0.44	0.03	0.74	0.04
		[0.36, 0.53]	[−0.04, 0.11]	[0.71, 0.78]	[−0.03, 0.11]
GBP/USD	Benchmark	0.28	0.02	0.62	0.04
		[0.20, 0.36]	[−0.06, 0.10]	[0.58, 0.69]	[−0.04, 0.12]
	UC-SV	0.34	0.03	0.64	0.04
		[0.26, 0.43]	[−0.04, 0.09]	[0.60, 0.69]	[−0.02, 0.10]
JPY/USD	Benchmark	0.44	0.00	0.88	0.03
		[0.37, 0.50]	[−0.08, 0.09]	[0.87, 0.92]	[−0.06, 0.11]
	UC-SV	0.47	0.00	0.89	0.02
		[0.35, 0.63]	[−0.06, 0.06]	[0.87, 0.92]	[−0.04, 0.08]
CAD/USD	Benchmark	0.56	0.04	0.66	0.06
		[0.51, 0.61]	[−0.05, 0.13]	[0.61, 0.74]	[−0.03, 0.14]
	UC-SV	0.57	0.05	0.66	0.06
		[0.45, 0.67]	[−0.02, 0.12]	[0.57, 0.74]	[−0.00, 0.13]
SEK/USD	Benchmark	0.52	0.04	0.72	0.05
		[0.46, 0.58]	[−0.03, 0.12]	[0.68, 0.79]	[−0.02, 0.13]
	UC-SV	0.57	0.05	0.75	0.06
		[0.51, 0.64]	[−0.01, 0.11]	[0.71, 0.79]	[0.00, 0.11]
CHF/USD	Benchmark	0.47	0.03	0.86	0.04
		[0.40, 0.53]	[−0.06, 0.11]	[0.84, 0.88]	[−0.04, 0.12]
	UC-SV	0.50	0.04	0.86	0.05
		[0.43, 0.57]	[−0.02, 0.10]	[0.85, 0.88]	[−0.01, 0.11]

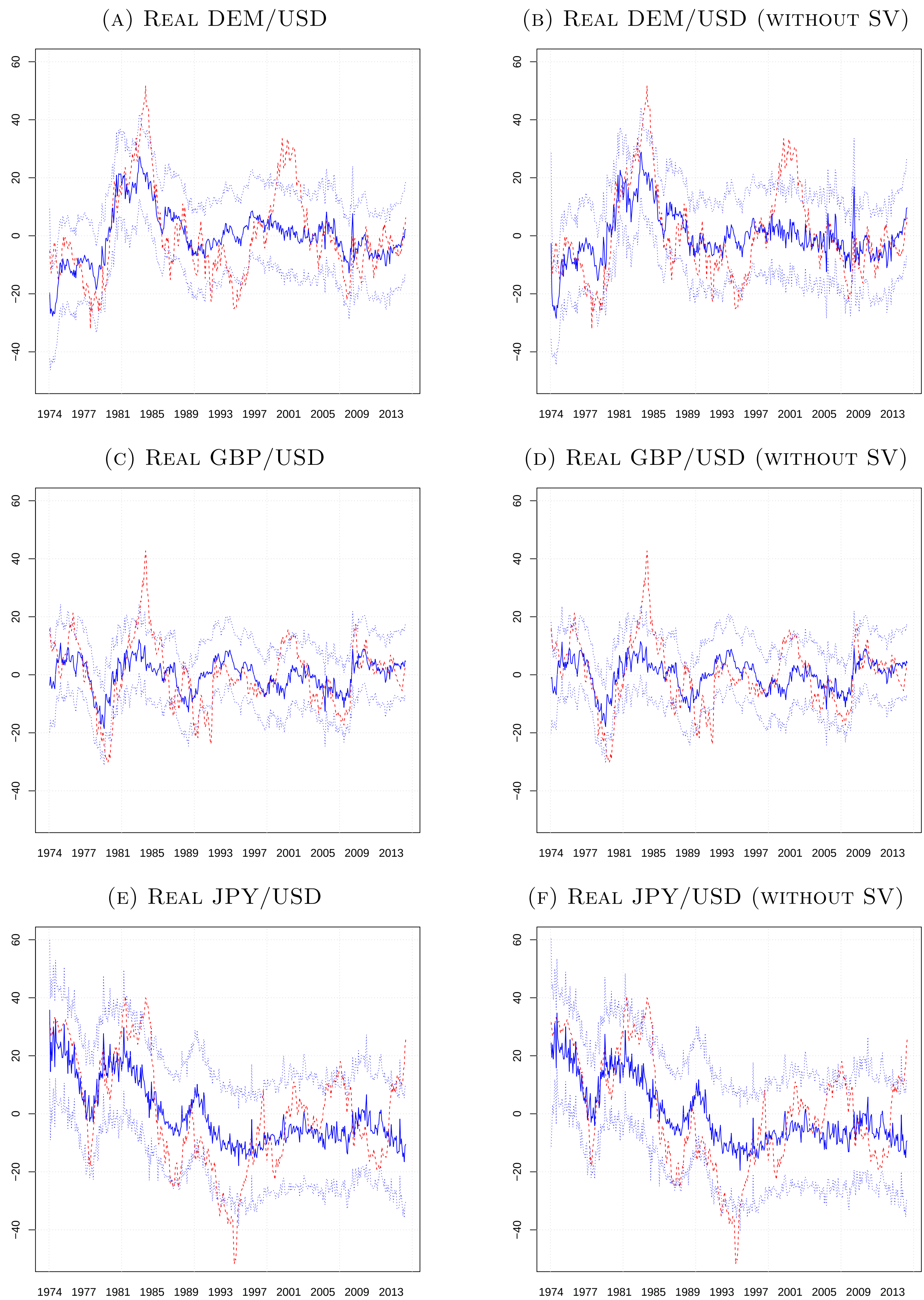
Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

Table 9 — Autocorrelation real exchange rate (without SV)

		Log-level			Log-change		
		1st	2nd	3rd	1st	2nd	3rd
DEM/USD	Actual	0.98	0.96	0.93	0.01	0.04	0.04
	Benchmark	0.94	0.90	0.88	−0.14	−0.22	−0.02
	UC-SV	0.93	0.86	0.83	−0.09	−0.22	−0.06
GBP/USD	Actual	0.97	0.94	0.90	0.33	0.02	0.05
	Benchmark	0.90	0.82	0.78	−0.12	−0.17	−0.06
	UC-SV	0.93	0.86	0.82	−0.08	−0.15	−0.07
JPY/USD	Actual	0.99	0.96	0.94	0.33	0.08	0.05
	Benchmark	0.90	0.89	0.87	−0.42	−0.02	−0.01
	UC-SV	0.93	0.91	0.90	−0.42	−0.04	−0.02
CAD/USD	Actual	0.99	0.98	0.97	0.21	0.05	0.03
	Benchmark	0.95	0.91	0.89	−0.16	−0.12	−0.10
	UC-SV	0.95	0.91	0.89	−0.13	−0.12	−0.11
SEK/USD	Actual	0.99	0.97	0.95	0.36	0.04	0.05
	Benchmark	0.86	0.76	0.73	−0.12	−0.26	−0.10
	UC-SV	0.91	0.83	0.80	−0.11	−0.24	−0.10
CHF/USD	Actual	0.98	0.96	0.93	0.27	0.03	0.02
	Benchmark	0.94	0.91	0.90	−0.18	−0.25	−0.03
	UC-SV	0.95	0.93	0.92	−0.13	−0.22	−0.03

Notes: Sample autocorrelation function for the actual real US Dollar exchange rate and sample autocorrelation function for the posterior mean of the model predictions up to 3rd order. The benchmark model does not take into account changes in the inflation and unemployment trends.

Figure 5 — Model predictions for large economies



(a) Real DEM/USD

(b) Real DEM/USD (without SV)

(c) Real GBP/USD

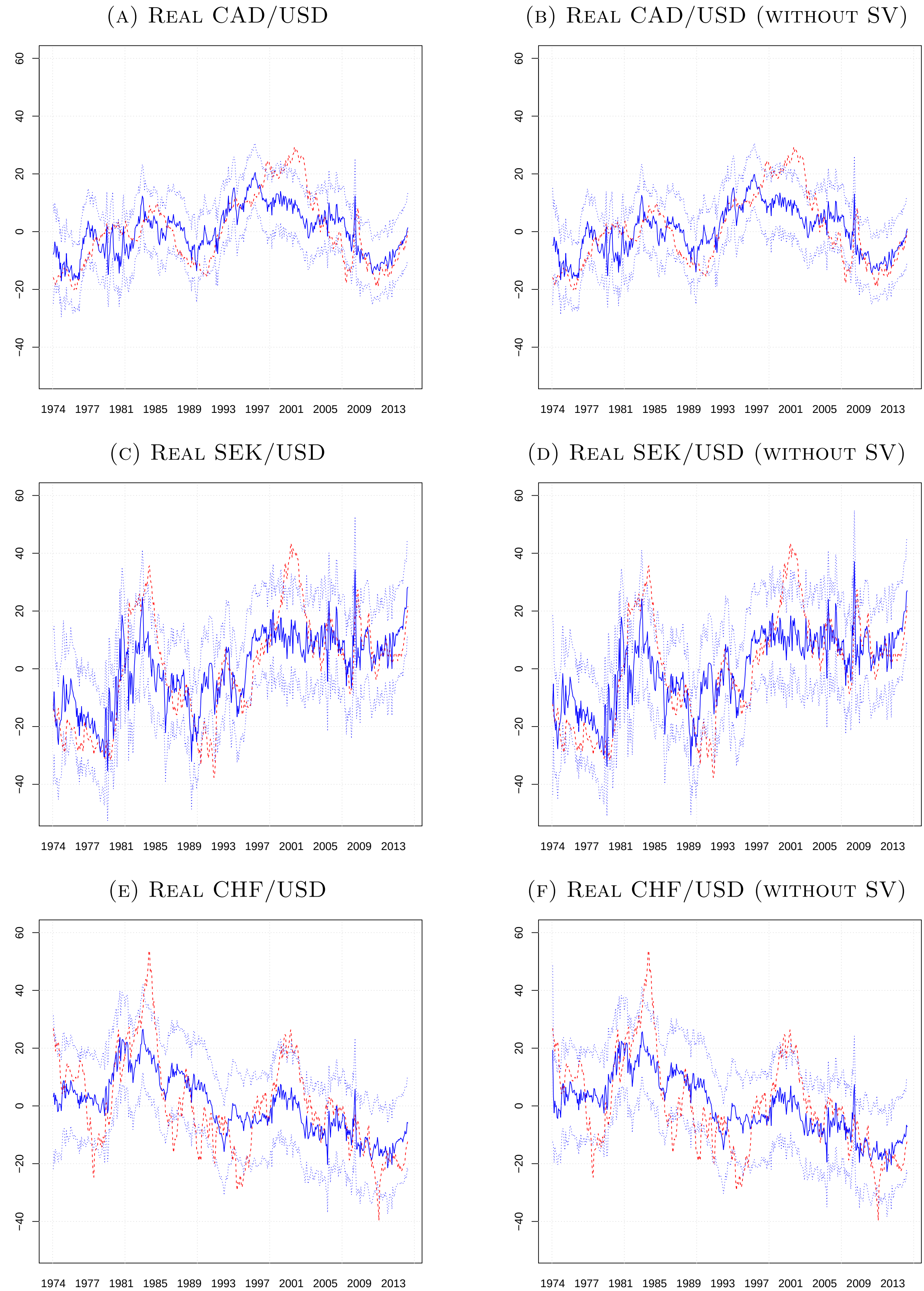
(d) Real GBP/USD (without SV)

(e) Real JPY/USD

(f) Real JPY/USD (without SV)

Notes: Actual real US Dollar exchange rates are given by dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

Figure 6 — Model predictions for small economies



Notes: Actual real US Dollar exchange rates in dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

Appendix C.2 Model with HP-filtered trend

TABLE 10 — CORRELATION WITH ACTUAL EXCHANGE RATE (HP-FILTER)

		(A) Real		(B) Nominal	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.42	0.02	0.73	0.03
		[0.35, 0.49]	[−0.09, 0.12]	[0.70, 0.80]	[−0.08, 0.13]
	UC-SV	0.55	0.00	0.78	0.01
		[0.51, 0.59]	[−0.07, 0.07]	[0.76, 0.80]	[−0.06, 0.08]
GBP/USD	Benchmark	0.28	0.02	0.62	0.04
		[0.20, 0.37]	[−0.06, 0.11]	[0.58, 0.71]	[−0.04, 0.12]
	UC-SV	0.44	0.02	0.68	0.03
		[0.40, 0.49]	[−0.04, 0.09]	[0.65, 0.71]	[−0.03, 0.09]
JPY/USD	Benchmark	0.44	0.00	0.88	0.02
		[0.37, 0.50]	[−0.08, 0.09]	[0.87, 0.89]	[−0.06, 0.11]
	UC-SV	0.38	−0.01	0.88	0.00
		[0.33, 0.43]	[−0.07, 0.05]	[0.87, 0.89]	[−0.06, 0.06]
CAD/USD	Benchmark	0.56	0.04	0.66	0.06
		[0.51, 0.61]	[−0.05, 0.13]	[0.62, 0.60]	[−0.03, 0.14]
	UC-SV	0.45	0.06	0.57	0.07
		[0.41, 0.50]	[−0.00, 0.13]	[0.53, 0.60]	[0.01, 0.13]
SEK/USD	Benchmark	0.52	0.04	0.72	0.05
		[0.46, 0.57]	[−0.03, 0.12]	[0.69, 0.82]	[−0.02, 0.13]
	UC-SV	0.66	0.04	0.81	0.06
		[0.63, 0.69]	[−0.01, 0.10]	[0.79, 0.82]	[0.00, 0.12]
CHF/USD	Benchmark	0.47	0.03	0.86	0.04
		[0.40, 0.53]	[−0.05, 0.12]	[0.84, 0.86]	[−0.04, 0.13]
	UC-SV	0.43	0.03	0.85	0.04
		[0.38, 0.47]	[−0.03, 0.09]	[0.83, 0.86]	[−0.02, 0.10]

Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

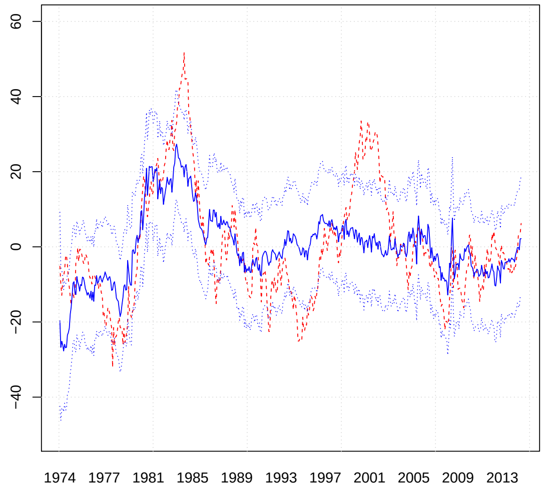
Table 11 — Autocorrelation real exchange rate (HP-filter)

		Log-level			Log-change		
		1st	2nd	3rd	1st	2nd	3rd
DEM/USD	Actual	0.98	0.96	0.93	0.01	0.04	0.04
	Benchmark	0.94	0.90	0.88	−0.14	−0.22	−0.03
	UC-SV	0.97	0.95	0.94	−0.32	−0.08	0.12
GBP/USD	Actual	0.97	0.94	0.90	0.33	0.02	0.05
	Benchmark	0.90	0.82	0.78	−0.13	−0.17	−0.06
	UC-SV	0.97	0.94	0.91	−0.03	−0.16	−0.05
JPY/USD	Actual	0.99	0.96	0.94	0.33	0.08	0.05
	Benchmark	0.90	0.89	0.87	−0.42	−0.01	−0.01
	UC-SV	0.94	0.92	0.91	−0.31	−0.16	0.02
CAD/USD	Actual	0.99	0.98	0.97	0.21	0.05	0.03
	Benchmark	0.95	0.91	0.89	−0.16	−0.12	−0.10
	UC-SV	0.93	0.86	0.82	−0.01	−0.17	−0.12
SEK/USD	Actual	0.99	0.97	0.95	0.36	0.04	0.05
	Benchmark	0.86	0.76	0.73	−0.12	−0.26	−0.10
	UC-SV	0.97	0.94	0.92	0.01	−0.17	−0.06
CHF/USD	Actual	0.98	0.96	0.93	0.27	0.03	0.02
	Benchmark	0.94	0.91	0.90	−0.18	−0.24	−0.04
	UC-SV	0.96	0.94	0.92	−0.13	−0.19	0.02

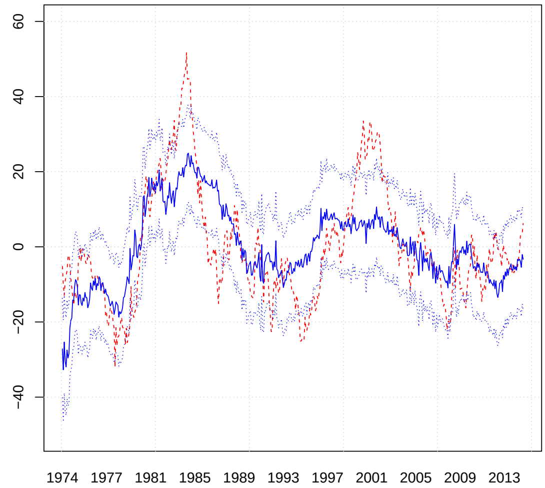
Notes: Sample autocorrelation function for the actual real US Dollar exchange rate and sample autocorrelation function for the posterior mean of the model predictions up to 3rd order. The benchmark model does not take into account changes in the inflation and unemployment trends.

Figure 7 — Model predictions for large economies

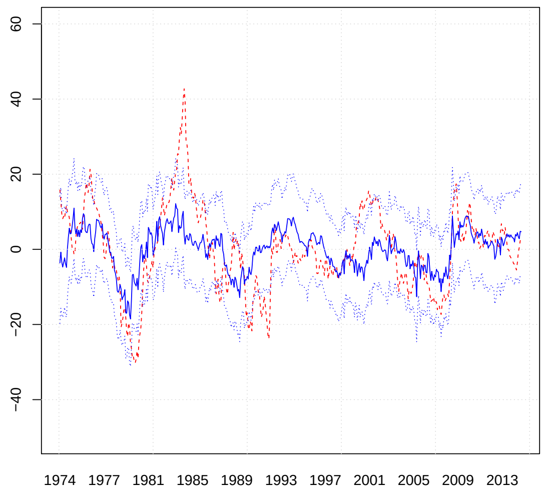
(a) Real DEM/USD



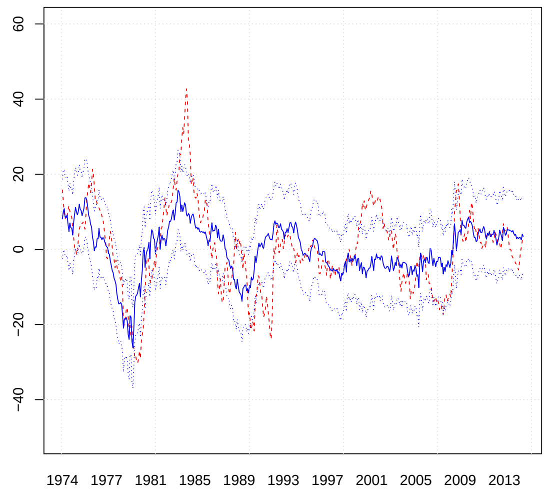
(b) Real DEM/USD (HP-filter)



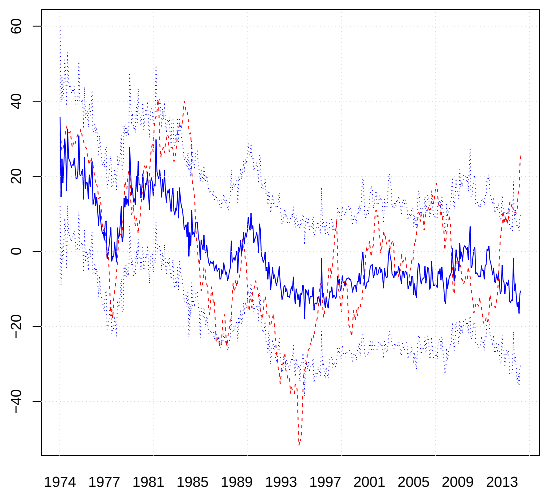
(c) Real GBP/USD



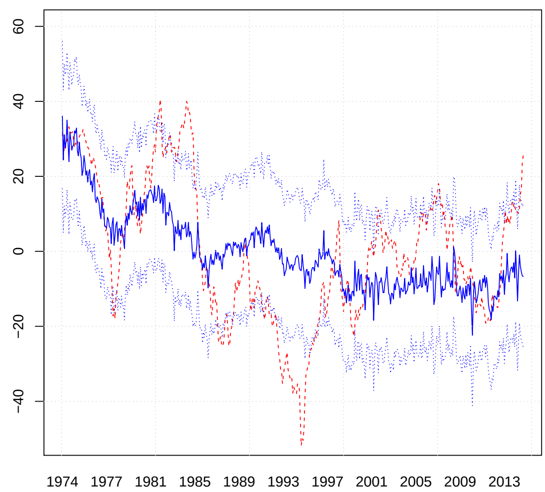
(d) Real GBP/USD (HP-filter)



(e) Real JPY/USD



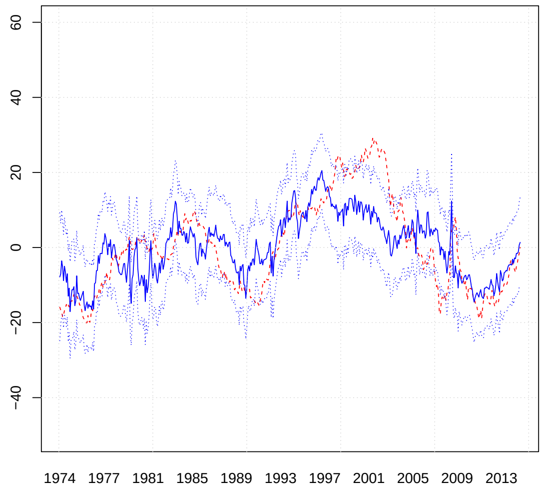
(f) Real JPY/USD (HP-filter)



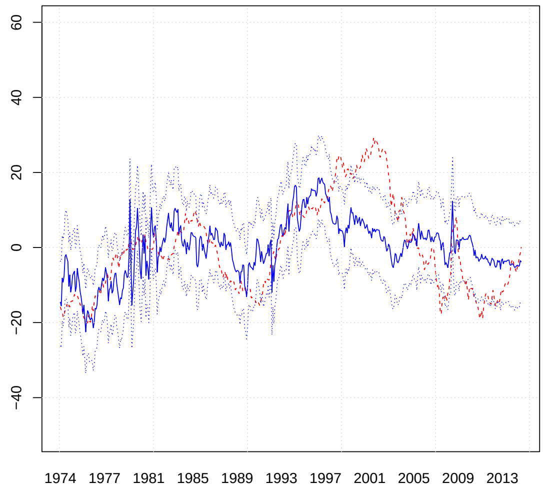
Notes: Actual real US Dollar exchange rates are given by dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

Figure 8 — Model predictions for small economies

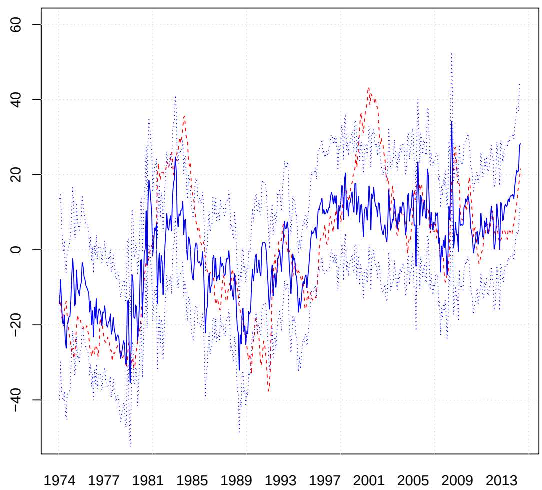
(a) Real CAD/USD



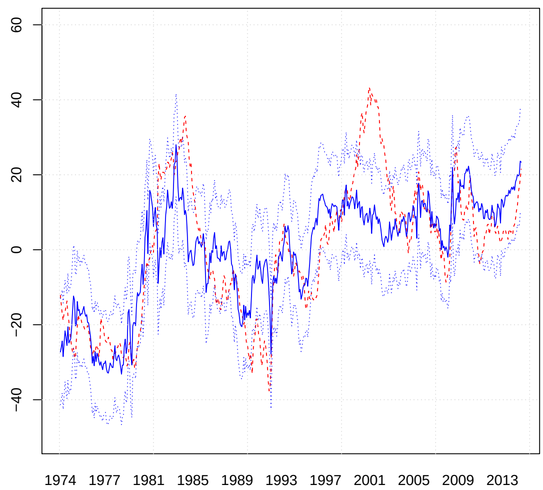
(b) Real CAD/USD (HP-filter)



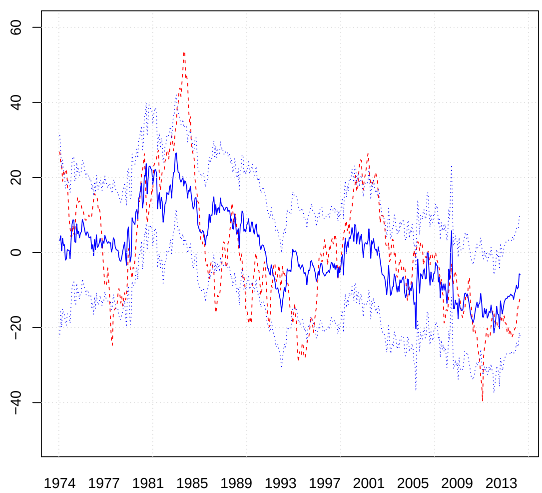
(c) Real SEK/USD



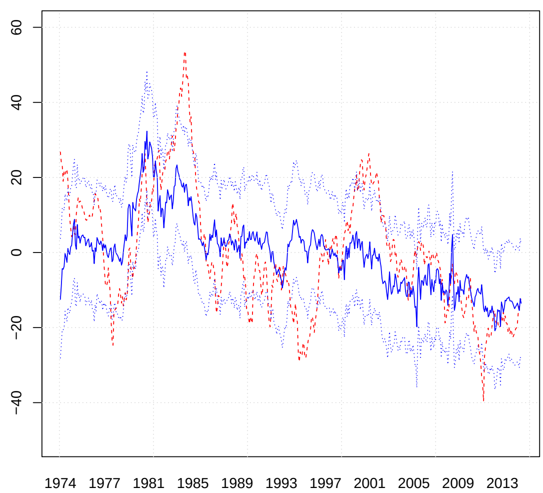
(d) Real SEK/USD (HP-filter)



(e) Real CHF/USD



(f) Real CHF/USD (HP-filter)



Notes: Actual real US Dollar exchange rates in dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

Appendix C.3 Model before introduction of Euro

Table 12 — Correlation with actual exchange rate (before introduction of Euro)

		(A) Real		(B) Nominal	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.66	0.02	0.80	0.02
		[0.61, 0.71]	[−0.11, 0.15]	[0.76, 0.86]	[−0.11, 0.15]
	UC-SV	0.70	0.03	0.82	0.04
		[0.64, 0.77]	[−0.06, 0.13]	[0.79, 0.86]	[−0.05, 0.13]

Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

Table 13 — Autocorrelation real exchange rate (before introduction of Euro)

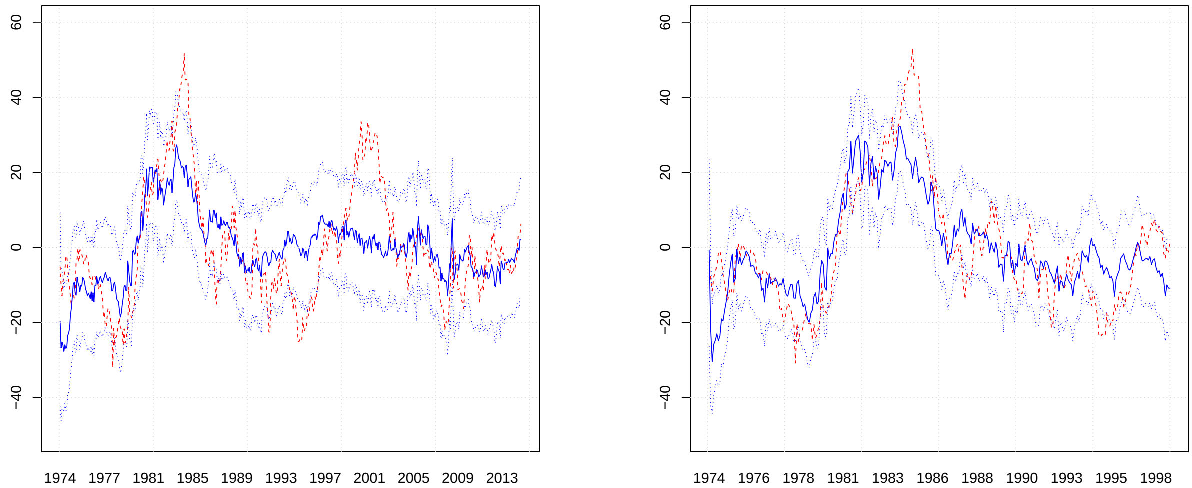
		(A) Real					
		Log-level			Log-change		
		1st	2nd	3rd	1st	2nd	3rd
DEM/USD	Actual	0.98	0.96	0.94	0.00	0.08	0.01
	Benchmark	0.96	0.93	0.91	−0.14	−0.13	0.00
	UC-SV	0.97	0.93	0.91	−0.04	−0.14	−0.04

Notes: Sample autocorrelation function for the actual real US Dollar exchange rate and sample autocorrelation function for the posterior mean of the model predictions up to 3rd order. The benchmark model does not take into account changes in the inflation and unemployment trends.

Figure 9 — Model predictions for large economies

(a) Real DEM/USD

(b) Real DEM/USD (before euro introduction)



Notes: Actual real US Dollar exchange rates are given by dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.